StudentID: ______________________

PHYS 222 Classical Mechanics II (Spring 2019)
Instructor: Sergiy Bubin
Diagnostic Exam

Instructions:

- The purpose of this test is to determine the level of student's current knowledge of mechanics and their mathematical skills. No score will be assigned for this test and it will have no effect on the final grade.
- No communication with classmates is allowed during the exam.
- This is a closed book exam. No notes, books, phones, tablets, calculators, etc. are allowed.
- Show all your work, explain your reasoning.
- Write legibly. If I cannot read and understand it then I will not be able to make a good judgement.
- Make sure pages are stapled together before submitting your work.

Problem 1. Given the following function of two variables x and y

$$f(x, y) = \sin(x + 2y)$$

find its Maclaurin series up to the terms of the third order.

Problem 2. Solve the following equation (α, β, and γ are constants):

$$\frac{d^2 f(t)}{dt^2} + 2\alpha \frac{df(t)}{dt} + \beta f(t) = 0$$

given that $f(0) = \gamma$ and $f'(0) = 0$.

Problem 3. Find the eigenvalues and normalized eigenvectors of the following matrix:

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

Problem 4. If $\psi(\mathbf{r})$ and $\mathbf{A}(\mathbf{r})$, where $\mathbf{r} = (x, y, z)$, are some differentiable scalar and vector functions respectively, simplify the following expressions. Make sure to write the result in a compact form.

(a) $\nabla \cdot (\psi \mathbf{A})$

(b) $\nabla^2 (\psi \mathbf{A})$

(c) $\nabla \cdot (\nabla \times \psi \mathbf{A})$

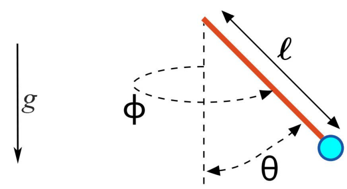
Problem 5. Evaluate the integral

$$I = \int_{-\infty}^{+\infty} e^{-\alpha x^2} dx, \qquad \alpha > 0$$

by noticing that $I^2 = I \cdot I = \int_{-\infty}^{+\infty} e^{-\alpha x^2} dx \cdot \int_{-\infty}^{+\infty} e^{-\alpha y^2} dy$ and making a transformation to the polar coordinates.

Problem 6. Consider a free particle, whose kinetic energy is small compared to its rest mass. Using the relativistic relation between the energy, momentum and rest mass, derive the leading correction to the nonrelativistic kinetic energy, $\frac{p^2}{2m}$.

Problem 7. A small ball of mass m is attached to the origin by a massless rod of length l and can rotate freely, subject to gravity. The position of this ball in space is determined by two angles, ϕ and θ.



(a) Write down the expressions for the kinetic and potential energies in terms of m, g, l, ϕ, θ, $\dot{\phi}$, and $\dot{\theta}$.

(b) Using the Lagrangian method, derive equations of motion for both ϕ and θ.

(c) Find an expression for the angular momentum L_z in terms of m, g, l, ϕ, θ, $\dot{\phi}$, and $\dot{\theta}$ (z axis is assumed to be in the direction of the gravity force).

(d) Rewrite the equations of motion for θ so that any mention of $\dot{\phi}$ is replaced by L_z.

(e) Consider a circular trajectory of constant $\theta = \pi/4$. What is L_z and what is the frequency of small oscillations of θ about $\pi/4$? Give answers in terms of m, g, l.