PHYS 222 Classical Mechanics II (Spring 2019)
Homework #1, due Thursday Jan 24 in class

Variational probems.

1. Consider the following functional

$$J[f(\mathbf{r})] = \int\int \frac{f(\mathbf{r}_1)f(\mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|} d\mathbf{r}_1 d\mathbf{r}_2,$$

   where $\mathbf{r}_i$ is a vector in 3D and the integration is over the entire space. Find the functional derivative $\frac{\delta J}{\delta f}$.

2. Find the second functional derivative, $\frac{\delta^2 F}{\delta f(x)\delta f(x')}$, for the functional that we used as one of the examples in lecture:

$$F[f(x)] = \int_{x_1}^{x_2} \left(f(x)\right)^{5/3} dx.$$

3. Consider the functional

$$F[y(x)] = \int_0^1 (y'^2 + 12xy) dx.$$

   Given that $y(0) = 0$ and $y(1) = 1$, find the curve $y(x)$ on which the functional is extemum.

4. Solve the problem that we did not have time to complete in the recitation: based on the Fermat's principle of least time derive the Snell's law of refraction at a boundary between two different isotropic media,

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{n_1}{n_2},$$

   where θ_i and n_i are the incident angle and index of refraction in medium i respectively.

5. Show that out of all figures of revolution (i.e. those that are axially symmetric) that have a given surface area S, a sphere maximizes the volume enclosed. *Hint: here you need to maximize a functional subject to a constraint.*