The potential energy is given by $V = -mgl\cos\theta$, while the kinetic energy in spherical coordinates is

$$T = \frac{1}{2}m(\dot{l}^2 + l^2\dot{\theta}^2 + l^2\sin^2\theta\,\dot{\phi}^2) = \frac{1}{2}ml^2(\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2)$$

because $\dot{l} = 0$ (the rod is rigid and its length is constant)

The Lagrangian of the system is then

$$L = T - V = \frac{1}{2}ml^2(\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2) + mgl\cos\theta$$

The generalized momenta can be easily calculated:

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = ml^2\sin^2\theta\,\dot{\phi} \quad \Rightarrow \quad \dot{\phi} = \frac{p_\phi}{ml^2\sin^2\theta}$$

$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = ml^2\dot{\theta} \quad \Rightarrow \quad \dot{\theta} = \frac{p_\theta}{ml^2}$$

With that we can write the Hamiltonian:

$$H = p_\phi\dot{\phi} + p_\theta\dot{\theta} - L(\phi, \theta, \dot{\phi}(p_\phi, \theta), \dot{\theta}(p_\theta)) =$$

$$= \frac{p_\phi^2}{ml^2\sin^2\theta} + \frac{p_\theta^2}{ml^2} - \frac{1}{2}ml^2\left(\frac{p_\theta^2}{m^2l^4} + \sin^2\theta\frac{p_\phi^2}{m^2l^4\sin^4\theta}\right) - mgl\cos\theta =$$

$$= \frac{1}{2ml^2}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right) - mgl\cos\theta$$