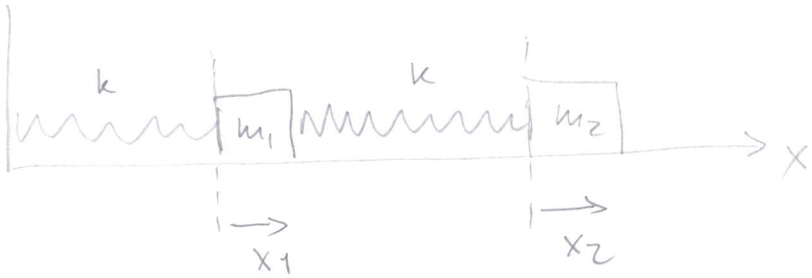
Let x_1 and x_2 be the coordinates measured from the position of equilibrium (i.e. when neither spring is stretched or compressed)



The Lagrangian of the system is

$$L = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} - \frac{k x_1^2}{2} - \frac{k(x_2 - x_1)^2}{2}$$

and the equations of motion are

$$\begin{cases} m\ddot{x}_1 + kx_1 - k(x_2 - x_1) = 0 \\ m\ddot{x}_2 + k(x_2 - x_1) = 0 \end{cases}$$

To determine the normal frequencies we use the ansatz

$$x_1 = a_1 e^{i\omega t} \qquad x_2 = a_2 e^{i\omega t} \qquad \left(\text{or, alternatively, } x_1 = a_1 \cos\omega t \quad x_2 = a_2 \cos\omega t \right)$$

The equations of motion then yield

$$\begin{aligned} (2k - m\omega^2) a_1 - k a_2 &= 0 \\ -k a_1 + (k - m\omega^2) a_2 &= 0 \end{aligned} \implies$$

Nontrivial solution is possible if

$$\begin{vmatrix} 2k - m\omega^2 & -k \\ -k & k - m\omega^2 \end{vmatrix} = 0$$

or

$$\omega^4 - 3\frac{k}{m}\omega^2 + \frac{k^2}{m^2} = 0 \implies \omega_{1,2}^2 = \frac{3\frac{k}{m} \pm \sqrt{5\frac{k^2}{m^2}}}{2}$$

The positive solutions are

$$\omega_1 = \sqrt{\frac{3+\sqrt{5}}{2}}\sqrt{\frac{k}{m}} = \frac{1}{2}\sqrt{1+5+2\sqrt{5}}\sqrt{\frac{k}{m}} = \frac{\sqrt{5}+1}{2}\sqrt{\frac{k}{m}}$$

$$\omega_2 = \sqrt{\frac{3-\sqrt{5}}{2}}\sqrt{\frac{k}{m}} = \frac{1}{2}\sqrt{1+5-2\sqrt{5}}\sqrt{\frac{k}{m}} = \frac{\sqrt{5}-1}{2}\sqrt{\frac{k}{m}}$$