a) The general solution for a vibrating string is a superposition of normal modes:

$$q(x,t) = \sum_n{}' (\beta_n \cos\omega_n t + \gamma_n \sin\omega_n t)\sin\left(\frac{n\pi x}{L}\right) \qquad \omega_n = \frac{n\pi}{L} v$$

Each mode is independent. Now, if $\dot{q}(x,0) = 0$ and $q(x,0) = C \sin\frac{3\pi x}{L}$ it is easy to see tha we start with $q(x,0)$ that contains only $n=3$ mode. That is $\beta_n \propto \delta_{n3}$ and $\gamma_n \propto \delta_{n3}$ ($\sin\frac{n\pi x}{L}$ is orthogonal to $\sin\frac{3\pi x}{L}$ unless $n=3$). Further

$$\beta_3 = C \qquad \text{and} \qquad \gamma_3 = 0$$

so the solution looks as follows

$$q(x,t) = C\cos\omega_3 t \sin\frac{3\pi x}{L} = C\cos\frac{3\pi v t}{L}\sin\frac{3\pi x}{L}$$

b) The initial shape of the string is antisymmetric with respect to the center point ($x = L/2$). Therefore, only the modes $n = 2, 4, 6, \ldots$ will contribute to the expansion of $q(x,t)$ in terms of the normal modes. The lowest and dominant normal frequency is then

$$\omega_2 = \frac{2\pi v}{L}$$