PHYS 451: Quantum Mechanics I
Homework #3, due Thursday September 11, in class

1. Consider a particle in an infinite square box of length a. Assume that initially the particle is in the ground state. Then suddenly, at time $t = 0^+$, the box is expanded instantaneously to the length of $2a$.

   (a) Find the coefficients of $\psi(x, t = 0^+)$ in the basis of the eigenstates of the new box (of length $2a$).

   (b) Will the system ever return to its initial state, and if so, at which time?

2. Consider the wavepacket:

$$\psi(x) = A \exp\left[ik_0 x - \frac{(x - x_0)^2}{4\sigma^2}\right],$$

   where A, k_0, x_0, and σ are some real constants.

   (a) Determine the normalization factor, A.

   (b) Find the wave function in the momentum space, $\psi(k)$.

   (c) Calculate $\langle x\rangle$, $\langle x^2\rangle$, and Δx.

   (d) Calculate $\langle p\rangle$, $\langle p^2\rangle$, and Δp.

   (e) Calculate the probability current, $j(x)$

3. Demonstrate that

$$\delta(x) = \lim_{\epsilon\to 0^+} \frac{1}{\pi} \frac{\epsilon}{x^2 + \epsilon^2}$$

   is a valid representation of the Dirac delta function. Namely, show that

   (a) $\int\limits_{-\infty}^{+\infty} \delta(x) f(x) dx = f(0)$ for any reasonably "nice" function $f(x)$.

   (b) $\delta(x) = \delta(-x)$.

   (c) $x\delta(x) = 0$.

   (d) $\delta(cx) = \frac{1}{|c|}\delta(x)$.

   (e) $\delta'(-x) = -\delta'(x)$.

   (f) $x\delta'(x) = -\delta(x)$.

Found an error or need a clarification? Email the instructor at sergiy.bubin@nu.edu.kz