PHYS 451: Quantum Mechanics I
Homework #4, due Thursday September 18, in class

1. Consider a particle in the following 1D potential:

$$V(x) = \begin{cases} x^2, & x \geq 0 \\ \infty, & x < 0 \end{cases}$$

Find the energy levels and eigenfunctions of this system.

2. Assuming that the kinetic energy operator is $\hat{T} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$ find the following commutators:
   (a) $[\hat{x}, \hat{p}_x]$
   (b) $[\hat{x}, \hat{p}_y]$
   (c) $[\hat{y}, \hat{p}_x]$
   (d) $[\hat{p}_x, \hat{p}_y]$
   (e) $[\hat{x}, \hat{T}]$
   (f) $[\hat{p}_x, \hat{T}]$

3. The orbital angular momentum operator is defined as

$$\hat{\mathbf{L}} = \hat{\mathbf{r}} \times \hat{\mathbf{p}} = \begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ \hat{x} & \hat{y} & \hat{z} \\ \hat{p}_x & \hat{p}_y & \hat{p}_z \end{vmatrix},$$

where $|...|$ stands for the determinant and $\mathbf{e}_i$ are unit vectors. Find commutators:
   (a) $[\hat{L}_x, \hat{L}_x]$
   (b) $[\hat{L}_x, \hat{L}_y]$

4. The ground state harmonic oscillator wave function in the coordinate space is $\psi(x) = \frac{\alpha^{1/4}}{\pi^{1/4}} e^{-\alpha x^2/2}$ (where $\alpha = \frac{m\omega}{\hbar}$). Find this wave function in the momentum space, $\tilde{\psi}(k)$ (i.e. find the Fourier transform of $\psi(x)$). Verify that $\tilde{\psi}(k)$ comes out properly normalized when the Fourier transform is defined in such a way that the factor is $\frac{1}{\sqrt{2\pi}}$.