PHYS 451: Quantum Mechanics I
Homework #5, due Thursday September 25, in class

1. Consider the following operators and determine whether they are linear or not:
   (a) Multiplication operator, $\hat{M}_a$: $\hat{M}_a f(x) = af(x)$ (a is a constant)

   (b) Exponentiation operator, $\hat{E}$: $\hat{E}f(x) = e^{f(x)}$

   (c) Differentiation operator, $\hat{D}_x$: $\hat{D}_x f(x) = \frac{\partial f(x)}{\partial x}$

   (d) Integration operator, $\hat{S}_x$: $\hat{S}_x f(x) = \int_0^x f(x')dx'$

   (e) Addition operator, $\hat{A}_a$: $\hat{A}_a f(x) = f(x) + a$ (a is a constant)

   (f) Translation operator, $\hat{T}_a$: $\hat{T}_a f(x) = f(x+a)$ (a is a constant)

   (g) Inversion operator, $\hat{I}$: $\hat{I}f(x) = f(-x)$

   (h) Complex conjugation operator, $\hat{C}$: $\hat{C}f(x) = f^*(x)$

   (i) Operator that projects out the odd part, $\hat{O}$: $\hat{O}f(x) = \frac{1}{2}[f(x) - f(-x)]$

   (j) Operator that computes the absolute value, $\hat{N}$: $\hat{N}f(x) = |f(x)|$

   (k) Operator that squares the argument, $\hat{Q}$: $\hat{Q}f(x) = f(x^2)$

2. Problem 3.4 in Griffiths.

3. Problem 3.5 in Griffiths.

4. Consider a particle that moves in 1D with Hamiltonian $\hat{H} = \frac{p^2}{2m} + V(x)$. Show that the uncertainties of Δp_x and ΔE obey the following inequality:

$$\Delta p_x \Delta E \geq \frac{\hbar}{2} \left| \left\langle \frac{\partial V}{\partial x} \right\rangle \right|.$$

   What does it imply for stationary states?

5. Show that the standard deviation $\Delta \hat{A} = \sqrt{\langle \hat{A}^2 \rangle - \langle \hat{A} \rangle^2}$ vanish only when the wave function is an eigenfunction of $\hat{A}$. (*Hint*: let $\psi = \sum_i c_i \psi_i$, where ψ_i are eigenfunctions of $\hat{A}$, i.e. $\hat{A}\psi_i = \alpha_i \psi_i$)

Found an error or need a clarification? Email the instructor at sergiy.bubin@nu.edu.kz