PHYS 451: Quantum Mechanics I
Homework #7, due Thursday October 23, in class

1. Show that an arbitrary operator $\hat{C}$ can be represented as sum $\hat{C} = \hat{A} + i\hat{B}$, where both $\hat{A}$ and $\hat{B}$ are hermitian operators.

2. Show that if operator $\hat{C}$ is hermitian then $\hat{B} = \hat{A}\hat{C}\hat{A}^\dagger$ is also a hermitian operator.

3. To exercise with the matrix form of quantum mechanical operators, compute the matrices of the position ($\hat{x}$), momentum ($\hat{p}$), and Hamiltonian ($\hat{H}$) operators in the basis of stationary states of the harmonic oscillator. In other words, for arbitrary values of quantum numbers n and k evaluate matrix elements $\langle n|\hat{x}|k\rangle$, $\langle n|\hat{p}|k\rangle$, and $\langle n|\hat{H}|k\rangle$, where $|n\rangle$ are the eigenfunctions of the harmonic oscillator Hamiltonian,

$$\hat{H} = -\frac{\hat{p}^2}{2m} + \frac{m\omega^2\hat{x}^2}{2}.$$

   Hint: The easiest way to do that is to use the lowering and raising operators (see lecture #5):

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\,\hat{x} + \frac{i\hat{p}}{\sqrt{2m\hbar\omega}} \ ,$$

$$\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\,\hat{x} - \frac{i\hat{p}}{\sqrt{2m\hbar\omega}} \ .$$

   First, express $\hat{x}$, $\hat{p}$, and $\hat{H}$ through $\hat{a}$ and $\hat{a}^\dagger$ and then evaluate the above matrix elements remembering that (again see lecture #5)

$$|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}\,|0\rangle \ ,$$

   and

$$[\hat{a}, \hat{a}^\dagger] = 1 \ .$$

   Write down the explicit form of (infinite) matrices $\hat{x}$, $\hat{p}$, and $\hat{H}$. The case of $\hat{H}$ is particularly simple. Can you guess how the matrix of $\hat{H}$ looks like in the basis of the eigenfunctions of $\hat{H}$ without doing any calculations?

4. Write all spherical harmonics up to $l = 2$ (there are nine of them) in Cartesian form, i.e. give expressions in terms of x, y, z, and r. You can either use the Rodrigues formula for the Legendre polynomials or start with the given expressions for Y_l^m in terms of θ and ϕ. In any event you must show your work.

Found an error or need a clarification? Email the instructor at sergiy.bubin@nu.edu.kz