PHYS 451: Quantum Mechanics I
Homework #10, due Thursday November 20, in class

1. Show that
$$\Delta\hat{L}_x\Delta\hat{L}_y \geq \frac{\hbar}{2}|\langle\hat{L}_z\rangle|$$
in a state with a definite value of l and m. For which values of l and m (if any) it becomes the *equality*. It might be helpful to review problem 2 in homework #8.

2. What are the Clebsch-Gordan coefficients involved in the expansion of the following states:
$$|2\,2\,1\,1\rangle, \quad |2\,1\,1\,1\rangle, \quad |2\,0\,1\,1\rangle, \quad |2\,{-1}\,1\,1\rangle, \quad |2\,{-2}\,1\,1\rangle \text{ ?}$$
Here $|l\,m\,l_1\,l_2\rangle$ stands for a state with a definite value of the total angular momentum (l) and its projection on the z-axis (m) formed by two particles that have orbital angular momenta l_1 and l_2.

   Hint: Start with state $|2\,2\,1\,1\rangle$ or $|2\,-2\,1\,1\rangle$. At some point you might want to use the raising or lowering operator, $\hat{L}_\pm = \hat{L}_{1\pm} + \hat{L}_{2\pm}$, to generate equations containing the unknown coefficients.

3. Problem 4.27 in Griffiths.

4. Consider a particle with spin 1/2.

   (a) Find the spin functions (i.e. spinors) that have a definite value of the spin projection (up or down) on an arbitrary axis. Assume that the axis is defined by a unit vector $\mathbf{n}$. Express your result (let us denote it $|\mathbf{n},\uparrow\rangle$ and $|\mathbf{n},\downarrow\rangle$) as a linear combination of the eigenstates of $\hat{\sigma}_z$ operator, $|\frac{1}{2}\rangle$ and $|-\frac{1}{2}\rangle$.

   (b) If the spin of the particle is oriented along the positive direction of the z-axis (i.e. the particle is in state $|\frac{1}{2}\rangle$) and the projection of the spin on axis $\mathbf{n}$ is measured, what is the probability of obtaining $+\frac{1}{2}$ value?

   Hint: You might consider operator $\mathbf{n}\cdot\boldsymbol{\sigma} = n_x\sigma_x + n_y\sigma_y + n_z\sigma_z$.

Found an error or need a clarification? Email the instructor at sergiy.bubin@nu.edu.kz