Quantum Mechanics I - Lecture 1

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Quantum mechanics: what is it?

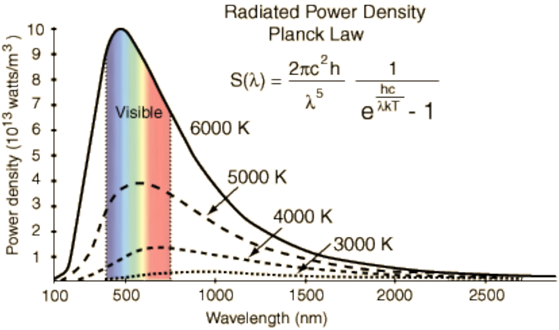
- One of the most important branches of physics
- Based on a set of fundamental definitions and equations
- Has a developed and powerful mathematical apparatus (Hilbert spaces, operators, probabilistic interpretation, etc.)

Why and when do we need quantum mechanics?

- Many observed phenomena cannot be descibed by classical physics
- QM is applicable when the action is of the order of Plank's constant
- QM is most often necessary to describe the microworld (though some macroscopic phenomena require quantum mechanics as well)

Timeline of quantum mechanics (most notable discoveries)

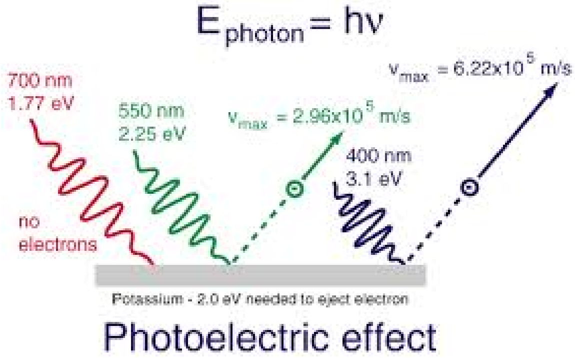
- **Max Planck (1901). Black body radiation.**
  Idea of quantized energy, or quants: $E = nh\nu$



$$\boxed{h = 6.62 \times 10^{34} \text{ J·s – Planck's constant}}$$

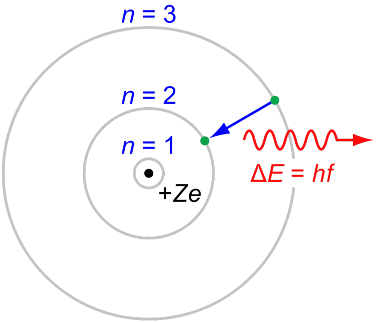
Timeline of quantum mechanics (most notable discoveries)

- **Albert Einstein (1905). Photoelectric effect.**
  $h\nu = W + \mathrm{K.E.}$



Timeline of quantum mechanics (most notable discoveries)

- **Niels Bohr (1913). Hydrogen atom model.**
  $\mu v r \equiv L = n\hbar$



$E_n = -\frac{\mu e^4}{2\hbar^2}\frac{Z^2}{n^2}$

- **Sommerfeld (1915). Extended Bohrs model to elliptical orbits.**

Timeline of quantum mechanics (most notable discoveries)

- **Louis de Broglie (1923). Particle-wave dualism.**
  $\boxed{p = \frac{h}{\lambda},\ E = h\nu}$
  $\mathbf{p} = \hbar\mathbf{k}$
- **Werner Heisenberg (1925). Matrix mechanics. Uncertainty principle**
  $\Delta x \Delta p_x \gtrsim \hbar$
- **Erwin Schrödinger (1926). Wave mechanics. The Schrödinger equation.**
  $i\hbar\frac{\partial\psi}{\partial t} = H\psi$
- **Paul Dirac (1927). Shows equivalence of the matrix and wave mechanics.**

The Schrödinger equation

Given the initial state, determines the time evolution of a quantum system

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi$$

$\psi(x, t)$ is the **wave function** (or the state vector) of the system.

- SE is postulated, not derived.
- The wave function provides the most complete description that can be given to a physical system.
- SE does not directly say what, exactly, the wave function is.

The Schrödinger equation: important properties

- Partial differential equation, 1st order in time, 2nd order in space coordinates.
- Linear. A sum of two solutions is a solution.
- Admits wave-like solutions if V does not depend on time explicitly. Hence, the SE can describe waves and is called a wave equation.

  $\xi = x - vt$

  $-i\hbar\frac{\partial}{\partial t}\psi(\xi) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi(\xi)$

  $i\hbar v\psi'(\xi) = -\frac{\hbar^2}{2m}\psi''(\xi) + V\psi(\xi)$

  The latter depends only on ξ, not x or t individually. Thus, one can indeed find wave-like solutions. They wind up looking like $e^{ik\xi}$.
- To solve the SE it is necessary to know the initial conditions (say $\psi(x, t = 0)$) and the "boundary" conditions (e.g. $\psi \to 0$ when $x \to \infty$).