Name: ____________________

PHYS 451: Quantum Mechanics I, Quiz #3

A particle is placed in the harmonic oscillator potential. Its initial wave function is

$$\Psi(x,0) = C[2\phi_0(x) - i\phi_1(x)],$$

where ϕ_0 and ϕ_1 are the eigefunctions of the the harmonic oscillator Hamiltonian corresponding to the ground and first excited states.

1. Is $\Psi(x,t)$ a stationary state? Explain briefly why.
2. Find the normalization constant C.
3. Write out $\Psi(x,t)$ and $|\Psi(x,t)|^2$. Make sure the latter is a real, nonnegative function.
4. Will the system ever return to its initial state, and if so, at what time?
5. Compute $\langle H\rangle$
6. Compute $\langle x\rangle$

Information that might be useful
Time-dependent Schrödinger equation: $i\hbar\frac{\partial\Psi}{\partial t} = H\Psi$
Stationary Schrödinger equation: $H\Psi = E\Psi$
The Hamiltonian of the harmonic oscillator: $H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{m\omega^2x^2}{2}$ (here m is the particle mass and ω is the angular frequency of the oscillator)
Energy levels of the harmonic oscillator: $E_n = \hbar\omega(n+\frac{1}{2}), \quad n = 0,1,2,...,\infty$
The explicit form of the lowest two eigenfunctions of the harmonic oscillator: $\phi_0(x) = \frac{\alpha^{1/4}}{\pi^{1/4}}\,e^{-\alpha x^2/2}$, $\phi_1(x) = \sqrt{2}\frac{\alpha^{3/4}}{\pi^{1/4}}\,x\,e^{-\alpha x^2/2}$, where $\alpha = \frac{m\omega}{\hbar}$
Gaussian integral: $\int\limits_{-\infty}^{+\infty} e^{-\beta x^2}dx = \sqrt{\frac{\pi}{\beta}} \quad (\beta > 0)$

Matrix elements of the position with the eigenfunctions of the harmonic oscillator: $\int\limits_{-\infty}^{+\infty}\phi_n^*(x)x\phi_k(x)dx = \sqrt{\frac{n}{2\alpha}}\delta_{n,k+1}$