PHYS 451: Quantum Mechanics I, Quiz #4

- 1. Determine if the following operators are hermitian, nonhermitian, or antihermitian (an operator \hat{O} is called antihermitian if $\hat{O}^{\dagger} = -\hat{O}$):
 - (a) Scaling operator: $\hat{S}_{\alpha}f(x) = \sqrt{\alpha}f(\alpha x)$ $(\alpha > 0)$
 - (b) $\hat{x}\hat{p}_x$
 - (c) $i(\hat{A}\hat{B} \hat{B}\hat{A})$, if it is known that $\hat{A}^{\dagger} = \hat{A}$ and $\hat{B}^{\dagger} = \hat{B}$
 - (d) $\hat{C} \hat{C}^{\dagger}$
 - (e) $\alpha \hat{x} \beta \frac{d}{dx}$ $(\alpha, \beta > 0)$

2. Consider a projection operator, \hat{P} , whose action on a wave function, $\psi(x)$, is defined as

$$\ddot{P}\psi = \gamma\phi,$$

where $\phi(x)$ is some given state and $\gamma = \langle \phi | \psi \rangle = \int \phi^*(x) \psi(x) dx$.

- (a) Is this operator linear?
- (b) Is it hermitian?