Name: ______________________

PHYS 451: Quantum Mechanics I, Quiz #5

Instruction: use additional sheets if you find it necessary

1. Prove the following operator identity:

$$e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{B} + [\hat{A},\hat{B}] + \frac{1}{2!}[\hat{A},[\hat{A},\hat{B}]] + ...$$

2. Suppose L^2 ($\hat{\mathbf{L}}$ is the angular momentum operator) is measured and the value $6\hbar^2$ is obtained. If L_y is measured immediately thereafter, what possible values can result?

3. What is the most probable value of r (irrespective of the direction of $\mathbf{r}$) in the ground state of the hydrogen atom?

4. A hydrogen atom starts out in the following linear combination of the stationary states $n = 2$, $l = 1$, $m = 1$ and $n = 2$, $l = 1$, $m = -1$:

$$\Psi(\mathbf{r}, t = 0) = \frac{1}{\sqrt{2}}(\psi_{211} + \psi_{21-1}).$$

   (a) Construct $\Psi(\mathbf{r}, t)$. Simplify it as much as you can.

   (b) Find the expectation value of the potential energy, $\langle V \rangle$.

5. Consider a system with the central potential $V(r) = -\frac{\hbar^2}{m}\frac{1}{r^2}$. Can you say something about its bound state energies corresponding to nonzero orbital angular momentum?

Appendix: Some information that may be useful

A function of an operator (say $\hat{C}$) can be defined by means of its Taylor series:

$$f(\hat{C}) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!}\hat{C}^k.$$

The equation for the radial component of the wave function, $R(r)$, of a particle moving in a spherically symmetric potential:

$$-\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial R}{\partial r} + \left[V(r) + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]R = ER.$$

The few first radial wave functions for hydrogen ($a = \frac{4\pi\epsilon_0\hbar^2}{me^2}$ is the Bohr radius):

$R_{10} = 2a^{-3/2}\, e^{-\frac{r}{a}}$

$R_{20} = \frac{1}{\sqrt{2}}a^{-3/2}\left(1 - \frac{1}{2}\frac{r}{a}\right)e^{-\frac{r}{2a}}$ $\qquad$ $R_{21} = \frac{1}{\sqrt{24}}a^{-3/2}\,\frac{r}{a}\,e^{-\frac{r}{2a}}$

The few first spherical harmonics:

$Y_0^0 = \left(\frac{1}{4\pi}\right)^{1/2}$

$Y_1^{-1} = \left(\frac{3}{8\pi}\right)^{1/2}\sin\theta\, e^{-i\phi}$ $\qquad$ $Y_1^0 = \left(\frac{3}{4\pi}\right)^{1/2}\cos\theta$ $\qquad$ $Y_1^1 = -\left(\frac{3}{8\pi}\right)^{1/2}\sin\theta\, e^{i\phi}$