PHYS 451: Quantum Mechanics I - Spring 2016
Homework #3, due February 9, in class

Schrödinger equation, particle in a box, quantum harmonic oscillator

1. Consider a particle of mass m whose wave function has the following form

$$\Psi(x,t) = \begin{cases} A\sin(\alpha x)e^{-\beta x - iEt/\hbar}, & x > 0 \\ 0, & x \leq 0 \end{cases}.$$

   where A, α, and β are some real positive constants. Find the energy of the particle, E, and the potential in which it moves.

2. Problem 2.4 in Griffiths.

3. A particle of mass m moves inside an infinite square well ($0 < x < a$). In this problem we will consider the time evolution of the system if the initial wave function is given by

$$\Psi(x, t = 0) = C(2\sin kx + 3\sin 2kx + \sin 3kx),$$

   where $k = \pi/a$.

   (a) Determine the normalization coefficient C.

   (b) Expand the wavefunction at the initial time in terms of the eigenfunctions $\phi_n(x)$ of the infinite square well, i.e. determine the coefficients c_n that define $\Psi(x, 0)$ as a superposition of eigenstates of the infinite square well.

   (c) Determine the time-dependent wave function, $\Psi(x,t)$.

   (d) Is the motion periodic? If so, what is the period?

   (e) If a measurement of the energy is performed, what will be the outcome(s) and, with what probability will those value(s) be measured?

   (f) What is the average energy of the particle? How does it change with time (assuming that no measurement is performed)?

   (g) Is the particle's energy changed if an energy measurement is performed? Comment on the energy conservation.

4. A particle of mass m moves in the potential

$$V(x) = V_1(x) + V_2(x),$$

   where $V_1(x) = \beta x$ and $V_2(x) = \gamma x^2$ (β and γ are some positive constants).

   (a) Find the lowest energy state of this potential, $V(x)$.

   (b) What is the probability that a particle starting out in the ground state of $V_2(x)$ only (i.e. when $V_1(x)$ is switched off) ends up in the new ground state of $V(x)$ when $V_1(x)$ is suddenly switched on?

5. Find the eigenvalues and eigenfunctions of the Hamiltonian with the potential

$$V(x) = \begin{cases} \frac{m\omega x^2}{2}, & x < 0 \\ \infty, & x \geq 0 \end{cases}$$

   Hint: Not much math is actually needed here. Some cleaver arguments can help you solve the problem relatively easily.