PHYS 451: Quantum Mechanics I - Spring 2016
Homework #6, due Friday April 8

Motion in 3D, spherically symmetric potentials, hydrogen atom

1. An electron is confined in an infinitely deep cubic potential well, whose sides are of length a and are parallel to the x, y, and z-axes.
   (a) Write the time-independent wave function corresponding to the states of the lowest and second lowest energy.
   (b) What is the degeneracy of energy levels for this system?
   (c) Find the number of states, N, that have energy less than some given E.

2. The motion of an electron is confined in a hollow spherical cavity of radius R. The walls of the cavity are impenetrable (i.e. this is an infinite spherical well). What is the pressure exerted on the walls of the cavity by the electron in its ground state?
   Hint: Recall the definition of the average force. In 1D it is given by $F_x = -\langle \frac{\partial V}{\partial x} \rangle$

3. A particle of mass m moves in a spherically symmetric potential in the form of an infinitely deep shell, i.e. between two concentric impermeable spheres of radii a and b ($b > a$). Find the ground state energy and normalized wave function.

4. Write all spherical harmonics up to $l = 2$ (there are nine of them) in Cartesian form, i.e. give expressions in terms of x, y, z, and $r = \sqrt{x^2 + y^2 + z^2}$. You can either use the Rodrigues formula for the Legendre polynomials or start with the given expressions for Y_l^m in terms of θ and ϕ. In any event you must show your work.

5. Consider a hydrogen atom. Its initial state is given by the wave function

$$\Psi(\mathbf{r}, t = 0) = \frac{1}{\sqrt{10}}\Big(2\psi_{100}(\mathbf{r}) + \psi_{210}(\mathbf{r}) + \sqrt{2}\psi_{211}(\mathbf{r}) + \sqrt{3}\psi_{21-1}(\mathbf{r})\Big),$$

   where the subscripts are the values of the quantum numbers n, l, and m.
   (a) Find the expectation value of the energy
   (b) Find the probability (as a function of time) that a measurement of $\mathbf{L}^2$ and L_z yields $2\hbar^2$ and $+\hbar$ respectively.
   (c) What is the probability of finding the electron within 1 Å of the proton at time $t = 0$? It is acceptable to make some good approximations here.
   (d) What is $\Psi(\mathbf{r}, t)$?

Found an error or need a clarification? Email the instructor at sergiy.bubin@nu.edu.kz