



$$\psi(x) = \begin{cases} h \left(1 - \frac{|x|}{a}\right), & -a \leq x \leq a \\ 0, & \text{otherwise} \end{cases}$$

Normalization:

$$\begin{aligned} \int_{-a}^a |\psi(x)|^2 dx &= 2 \int_0^a |\psi|^2 dx = 2h^2 \int_0^a \left(1 - \frac{x}{a}\right)^2 dx = 2h^2 \int_0^a \left(1 - \frac{2}{a}x + \frac{1}{a^2}x^2\right) dx = \\ &= 2h^2 \left(a - a + \frac{a^3}{3a^2}\right) = \frac{2}{3}h^2 a \Rightarrow h^2 = \frac{3}{2a} \quad h = \sqrt{\frac{3}{2a}} \end{aligned}$$

b)  $\langle x \rangle = 0$  due to symmetry (odd integrand)

$$\begin{aligned} \langle x^2 \rangle &= 2 \int_0^a x^2 |\psi|^2 dx = 2h^2 \int_0^a x^2 \left(1 - \frac{x}{a}\right)^2 dx = \frac{3}{a} \int_0^a \left(x^2 - \frac{2}{a}x^3 + \frac{1}{a^2}x^4\right) dx = \\ &= \frac{3}{a} \left(\frac{a^3}{3} - \frac{a^3}{2} + \frac{a^3}{5}\right) = 3a^2 \left(\frac{10 - 15 + 6}{30}\right) = \frac{a^2}{10} \end{aligned}$$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \frac{a}{\sqrt{10}}$$

c)  $P(-\Delta x \leq x \leq \Delta x) = \int_{-\Delta x}^{\Delta x} |\psi|^2 dx = 2 \int_0^{\Delta x} |\psi|^2 dx = 2h^2 \int_0^{\frac{a}{\sqrt{10}}} \left(1 - \frac{2}{a}x + \frac{1}{a^2}x^2\right) dx =$

$$= \frac{3}{a} \left(\frac{a}{\sqrt{10}} - \frac{a}{10} + \frac{a}{3 \cdot 10 \sqrt{10}}\right) = 3 \frac{30 - 3\sqrt{10} + 1}{30\sqrt{10}} = \frac{30 - 3\sqrt{10} + 1}{10\sqrt{10}} \approx 0.68$$