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PHYS 451: Quantum Mechanics I – Spring 2016
Quiz #2

Consider a particle moving in the infinite square well potential ($0 \leq x \leq a$). Suppose the initial state of the particle is given by

$$\Psi(x,0) = A[2\phi_1(x) - i\phi_2(x)],$$

where ϕ_1 and ϕ_2 are eigefunctions of the Hamiltonian corresponding to the ground and first excited states respectively.

(a) Find the normalization constant A.

(b) Write out $\Psi(x,t)$ and $|\Psi(x,t)|^2$. Make sure the latter is a real, nonnegative function, i.e. write your answer in such a form that the nonnegativity and abscence of any imaginary part can be seen easily.

(c) Is $\Psi(x,t)$ a stationary state?

(d) If you measure the energy at some given time t in a large ensemble of such particles, what will be the average value of your measurement?

(e) If you measure the position at some given time t in a large ensemble of such particles, what will be the average value of your measurement?

Appendix: Some information that might be useful

The Schrödinger equation

Time-dependent: $i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi$ Stationary: $\hat{H}\phi_n = E_n\phi_n$

Infinite square well ($0 \leq x \leq a$)

Energy levels: $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$, $n = 1, 2, ..., \infty$ or $E_n = n^2\hbar\omega$, where $\omega \equiv \frac{\pi^2\hbar}{2ma^2}$

Eigenfunctions: $\phi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$

Matrix elements of the position: $\int_0^a \phi_n^*(x)x\,\phi_k(x)dx = \begin{cases} a/2, & n = k \\ 0, & n \neq k;\ n \pm k \text{ is even} \\ -\frac{8nka}{\pi^2(n^2-k^2)^2}, & n \neq k;\ n \pm k \text{ is odd} \end{cases}$