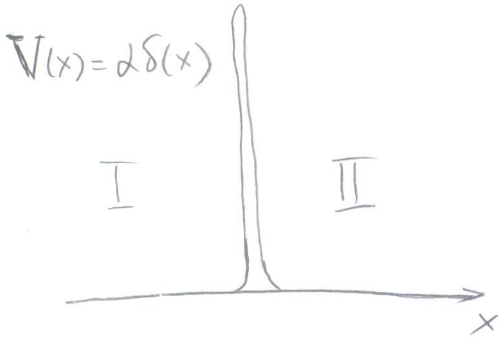

a) If we take the Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

and integrate it from $-\epsilon$ to ϵ (where ϵ is an infinitely small parameter)

we get:

$$\underbrace{-\frac{\hbar^2}{2m}\int_{-\epsilon}^{\epsilon}\frac{d^2\psi}{dx^2}\,dx}_{-\frac{\hbar^2}{2m}\left(\psi'(\epsilon)-\psi'(-\epsilon)\right)} + \underbrace{\alpha\int_{-\epsilon}^{\epsilon}\delta(x)\psi(x)\,dx}_{\alpha\psi(0)} = \underbrace{E\int_{-\epsilon}^{\epsilon}\psi(x)\,dx}_{0}$$

or

$$\psi'(\epsilon) - \psi'(-\epsilon) = \frac{2m\alpha}{\hbar^2}\psi(0)$$

b) In region I $(x<0)$ the Schrödinger equation is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \quad \text{or} \quad \psi'' + k^2\psi = 0 \quad \text{where } k = \frac{\sqrt{2mE}}{\hbar}$$

and the general solution of this differential equation is

$$\psi_I = Ae^{ikx} + Be^{-ikx}$$

Similarly, in region II we have

$$\psi_{II} = Ce^{ikx} + De^{-ikx}$$

If we are to compute the transmission probability we must assume $D=0$ since there is no incoming wave (e^{-ikx}) from the right. Then

$$\psi = \begin{cases} Ae^{ikx} + Be^{-ikx}, & x<0 \\ Ce^{ikx}, & x>0 \end{cases} \qquad \psi' = \begin{cases} ik(Ae^{ikx} - Be^{-ikx}), & x<0 \\ ikCe^{ikx}, & x>0 \end{cases}$$

The continuity of the wave function at $x=0$ gives

$$A + B = C \qquad (*)$$

while for the derivative of ψ we have, according to part a)

$$ikC - ik(A-B) = \frac{2m\alpha}{\hbar^2} C \qquad (**)$$

The transmission probability is given by the ratio

$$T = \frac{|C|^2}{|A|^2}$$

Solving equation (*) for B and substituting it into equation (**) yields

$$B = C - A \qquad C - A + C - A = -\frac{2im\alpha}{\hbar^2 k} C$$

$$A = C\left(1 + \frac{im\alpha}{\hbar^2 k}\right)$$

$$T = \frac{1}{1 + \frac{m^2\alpha^2}{\hbar^4 k^2}} = \frac{1}{1 + \frac{m\alpha^2}{2\hbar^2 E}}$$