

1. a) No, because $[x, p] \neq 0$ and correspondingly $\Delta x \Delta p \neq 0$

b) Generally no, because $[x, \frac{p^2}{2m}] \neq 0$ and correspondingly $\Delta x \Delta(\frac{p^2}{2m}) \neq 0$ (unless the state is such that $\langle [x, \frac{p^2}{2m}] \rangle = 0$)

c) Yes, because $[x, V(x)] = 0$ and correspondingly $\Delta x \Delta V = 0$

d) Generally no, because $[x, H] = [x, \frac{p^2}{2m} + V] \neq 0$, so $\Delta x \Delta E \neq 0$ (unless, again, $\langle [x, \frac{p^2}{2m}] \rangle = 0$)

2. Let us use the general expression for the uncertainty principle, namely $\Delta x \Delta E \geq \frac{1}{2} |\langle [x, H] \rangle|$

For the commutator we have,

$$\begin{aligned} [x, H] f &= [x, \frac{p^2}{2m}] f = -\frac{\hbar^2}{2m} [x, \frac{d^2}{dx^2}] f = -\frac{\hbar^2}{2m} (x \frac{d^2 f}{dx^2} - \frac{d^2}{dx^2} x f) \\ &= -\frac{\hbar^2}{2m} (x f'' - \frac{d}{dx} (f + x f')) = -\frac{\hbar^2}{2m} (x f'' - f' - f' - x f'') = \frac{\hbar^2}{m} f' \end{aligned}$$

$$\text{or } [x, H] = \frac{\hbar^2}{m} \frac{d}{dx} = \frac{i\hbar}{m} p$$

Then the uncertainty principle reads

$$\Delta x \Delta E \geq \frac{\hbar}{2m} |\langle p \rangle|$$

Now, for stationary states the energy is definite, so $\Delta E = 0$. From here it follows that $\langle p \rangle$ must be zero:

$$\langle p \rangle = 0$$

which makes perfect sense since nothing is really moving in a stationary state.