The basis states $|sm\rangle$ (eigenfunctions of S^2 and S_z) are

$$|\tfrac{3}{2},\tfrac{3}{2}\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad |\tfrac{3}{2},\tfrac{1}{2}\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad |\tfrac{3}{2},-\tfrac{1}{2}\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad |\tfrac{3}{2},-\tfrac{3}{2}\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

The action of S_+ and S_- on states $|\tfrac{3}{2},m\rangle$ is the following: $S_\pm|\tfrac{3}{2},m\rangle = \hbar\sqrt{\tfrac{15}{4} - m(m\pm1)}\,|\tfrac{3}{2},m\pm1\rangle$ or, separately for each m:

$$S_+|\tfrac{3}{2},\tfrac{3}{2}\rangle = 0 \qquad S_+|\tfrac{3}{2},\tfrac{1}{2}\rangle = \sqrt{3}\hbar\,|\tfrac{3}{2},\tfrac{3}{2}\rangle$$

$$S_+|\tfrac{3}{2},-\tfrac{1}{2}\rangle = 2\hbar\,|\tfrac{3}{2},\tfrac{1}{2}\rangle \qquad S_+|\tfrac{3}{2},-\tfrac{3}{2}\rangle = \sqrt{3}\hbar\,|\tfrac{3}{2},-\tfrac{1}{2}\rangle$$

and

$$S_-|\tfrac{3}{2},\tfrac{3}{2}\rangle = \sqrt{3}\hbar\,|\tfrac{3}{2},\tfrac{1}{2}\rangle \quad S_-|\tfrac{3}{2},\tfrac{1}{2}\rangle = 2\hbar\,|\tfrac{3}{2},-\tfrac{1}{2}\rangle \quad S_-|\tfrac{3}{2},-\tfrac{1}{2}\rangle = \sqrt{3}\hbar\,|\tfrac{3}{2},-\tfrac{3}{2}\rangle$$

$$S_-|\tfrac{3}{2},-\tfrac{3}{2}\rangle = 0$$

Thus

$$S_+\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} = \begin{pmatrix}0&\sqrt{3}\hbar&0&0\\0&0&2\hbar&0\\0&0&0&\sqrt{3}\hbar\\0&0&0&0\end{pmatrix} \Rightarrow S_+ = \hbar\begin{pmatrix}0&\sqrt{3}&0&0\\0&0&2&0\\0&0&0&\sqrt{3}\\0&0&0&0\end{pmatrix}$$

$$S_-\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} = \begin{pmatrix}0&0&0&0\\\sqrt{3}\hbar&0&0&0\\0&2\hbar&0&0\\0&0&\sqrt{3}\hbar&0\end{pmatrix} \Rightarrow S_- = \hbar\begin{pmatrix}0&0&0&0\\\sqrt{3}&0&0&0\\0&2&0&0\\0&0&\sqrt{3}&0\end{pmatrix}$$

Finally, $S_x = \frac{1}{2}(S_+ + S_-) = \hbar\begin{pmatrix}0&\frac{\sqrt{3}}{2}&0&0\\\frac{\sqrt{3}}{2}&0&1&0\\0&1&0&\frac{\sqrt{3}}{2}\\0&0&\frac{\sqrt{3}}{2}&0\end{pmatrix}$

The eigenvalues of S_x are $+\frac{3}{2}\hbar, +\frac{1}{2}\hbar, -\frac{1}{2}\hbar, -\frac{3}{2}\hbar$. This can be verified by a direct calculation. However, we can also determine that based on isotrophy: x-direction is no better or worse than the x-direction. Thus possible projections must be the same