PHYS 451: Quantum Mechanics I - Spring 2017
Homework #4, due Tuesday February 14 in class

Fourier transform, Dirac delta function, reflection and transmission coefficients

1. Consider the wave function that has the following form in the coordinate space,

$$\psi(x) = Ae^{-\beta(x-a)^2},$$

   where β and a are some positive constants.

   (a) Find the normalization factor A.

   (b) Compute the Fourier transfom of this function, $\tilde{\psi}(k)$. Please do the math by yourself in this part (i.e. do not refer to any tables of integrals). $\tilde{\psi}(k)$ is the wave function in the momentum space. Show that it comes out normalized in the k-space.

   (c) Compute the expectation values $\langle p \rangle$ and $\langle p^2 \rangle$ the usual way, i.e. using the wave function in the coordinate space.

   (d) Keeping in mind the de Broglie relation ($p = \hbar k$), repeat the same exercise in the momentum space, i.e. use $\tilde{\psi}(k)$ to compute $\langle p \rangle$ and $\langle p^2 \rangle$.

2. Consider the following representation of the Dirac delta function:

$$g(x) = \lim_{\epsilon \to 0^+} \frac{1}{\sqrt{\pi}\epsilon} \exp\left[-\frac{x^2}{\epsilon^2}\right].$$

   Show that this representation satisfies the basic properties of the delta function

   (a) $\int\limits_{-\infty}^{+\infty} g(x)f(x)dx = f(0)$ for any reasonably "nice" function $f(x)$

   (b) $g(x) = g(-x)$

   (c) $xg(x) = 0$

   (d) $g(\alpha x) = \frac{1}{|\alpha|}g(x)$ (here α is a real constant)

   (e) $g'(-x) = -g'(x)$

   (f) $xg'(x) = -g(x)$

   (g) $\tilde{g}(k) = \frac{1}{\sqrt{2\pi}}$ (Fourier transform)

3. Problem 2.44 in Griffiths

4. Find the transmission and reflection coefficients for the potential in the form of a step:

$$V(x) = \begin{cases} 0 & , x < 0 \\ V_0 & , x \geq 0 \end{cases},$$

   where $V_0 > 0$. Assume that the particle is incident from the left and the energy of the particle is greater than V_0. Examine the limiting cases $E \to V_0$ and $E \to \infty$.