PHYS 451: Quantum Mechanics I - Spring 2017
Homework #5, due Thursday February 23 in class

Creation and annnihilation operators, commutation relations, the uncertainty principle, matrix formalism

1. Using the formalism of the creation and annihilation operators ($a^\dagger$ and a) compute the following general matrix elements in the basis of harmonic oscillator functions $\psi_n(x)$:

   (a) $\langle \psi_n | x | \psi_m \rangle$

   (b) $\langle \psi_n | x^2 | \psi_m \rangle$

   (c) $\langle \psi_n | p | \psi_m \rangle$

   (d) $\langle \psi_n | p^2 | \psi_m \rangle$

   Then find how the uncertainty principle holds for state n, i.e.

   (e) compute $\Delta x \Delta p$ for state ψ_n.

   Hint: first express x and p in terms of $a^\dagger$ and a, then recall from lecture how $a^\dagger$ and a act on the eigenfunctions of the Hamiltonian.

2. Consider two $n \times n$ matrices P and X (n is a *finite* number). Can they satisfy the canonical commutation relation
$$[P, X] = -i\hbar I,$$
   where I is the identity matrix? If so, give an example how these matrices may look like.

3. Consider a particle that moves in 1D with Hamiltonian $\hat{H} = \frac{p^2}{2m} + V(x)$.

   (a) Show that the uncertainties of Δp_x and ΔE obey the following inequality:
$$\Delta p_x \Delta E \geq \frac{\hbar}{2} \left| \left\langle \frac{\partial V}{\partial x} \right\rangle \right|.$$

   (b) What does it imply for stationary states?

4. Problem 3.38 in Griffiths