PHYS 451: Quantum Mechanics I - Spring 2017
Homework #6, due Thursday March 2 in class

General properties of operators, spherical harmonics

1. Assuming that a function of an operator is defined as the corresponding Taylor series, i.e. $F(\hat{A}) = \sum\limits_{n=0}^{\infty} \frac{F^{(n)}(0)}{n!} \hat{A}^n$, determine the explicit form of the following operators (or you may write how they act on an arbitrary function $f(x)$):

   (a) $\exp(i\gamma\hat{I})$

   (b) $\exp\left(\frac{ia\hat{p}}{\hbar}\right)$

   (c) $\exp\left(bx\frac{d}{dx}\right)$

   In the above expressions $\hat{I}$ is the inversion operator (i.e. $\hat{I}f(x) = f(-x)$), $\hat{p}$ is the momentum operator, while γ, a, and b are some constants.

2. Prove the following operator relation: $e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{B} + [\hat{A}, \hat{B}] + \frac{1}{2!}[\hat{A}, [\hat{A}, \hat{B}]] + \ldots$

3. Assuming that λ is a small parameter, find an expansion of the operator $(\hat{A} - \lambda\hat{B})^{-1}$ in the powers of λ.

4. Write all spherical harmonics up to $l = 2$ (there are nine of them) in Cartesian form, i.e. give expressions in terms of x, y, z, and $r = \sqrt{x^2 + y^2 + z^2}$. You can either use the Rodrigues formula for the Legendre polynomials or start with the given expressions for Y_l^m in terms of θ and ϕ. In any event you must show your work.