PHYS 451: Quantum Mechanics I - Spring 2017
Homework #7, due Thursday March 9 in class

Motion in 3D, central potentials

1. An electron is confined in an infinitely deep cubic potential well, whose sides are of length a and are parallel to the x, y, and z-axes.

   (a) Write the time-independent wave function corresponding to the states of the lowest and second lowest energy.

   (b) What is the degeneracy of energy levels for this system?

   (c) Find the number of states, N, that have energy less than some given E.

2. Consider a particle of mass m that is constrained to move in between two concentric impenetrable spheres of radius a and b. In other words, the particle moves in the following central potential:

$$V(r) = \begin{cases} 0, & a < r < b \\ \infty, & \text{otherwise} \end{cases}$$

   Find the ground state energy and wave function.

3. Problem 4.13 in Griffiths.

4. Consider a hydrogen atom. Its initial state is given by the wave function

$$\Psi(\mathbf{r}, t = 0) = \frac{1}{\sqrt{10}}\Big(2\psi_{100}(\mathbf{r}) + \psi_{210}(\mathbf{r}) + \sqrt{2}\psi_{211}(\mathbf{r}) + \sqrt{3}\psi_{21-1}(\mathbf{r})\Big),$$

   where the subscripts are the values of the quantum numbers n, l, and m.

   (a) Find the expectation value of the energy

   (b) Find the probability (as a function of time) that a measurement of $\mathbf{L}^2$ and L_z yields $2\hbar^2$ and $+\hbar$ respectively.

   (c) What is the probability of finding the electron within 1 Å of the proton at time $t = 0$? You can make some reasonable approximations here if you want to.

   (d) What is $\Psi(\mathbf{r}, t)$?