PHYS 451: Quantum Mechanics I - Spring 2017
Homework #8, due Tuesday March 28 in class

Angular momentum

1. State $|\psi\rangle$ is an eigenstate of $\hat{\mathbf{L}}^2$ and $\hat{L}_z$, i.e.

$$\hat{\mathbf{L}}^2|\psi\rangle = \hbar^2 l(l+1)|\psi\rangle \quad \text{and} \quad \hat{L}_z|\psi\rangle = \hbar m|\psi\rangle$$

Find $\langle \hat{L}_x \rangle$ and $\langle \hat{L}_x^2 \rangle$ in this state.
Hint: it may be helpful to take into account the symmetry with respect to x and y.

2. A spinless particle has the following wave function:

$$\psi = A(x+y+2z)e^{-\beta r},$$

where A and β are positive constants and $r = \sqrt{x^2+y^2+z^2}$.

   (a) What is the total angular momentum of the particle?

   (b) What is the expectation value of the z-component of the angular momentum?

   (c) What are the probabilities of getting $+2\hbar$ and $+\hbar$ and 0 upon measuring the z-component of the angular momentum?

   (d) What is the probability of finding the particle at angles θ and ϕ (azimuthal and polar angle respectively) in solid angle $d\Omega$?

3. The operator describing a rotation around the y-axis by $\pi/2$ has the form $\hat{R}_y(\pi/2) = e^{-i\frac{\pi}{2}\frac{\hat{L}_y}{\hbar}}$. Prove the rotation operator relation

$$\hat{R}_y(-\pi/2)\,\hat{L}_z\,\hat{R}_y(\pi/2) = -\hat{L}_x.$$

Now generalize this result for an arbitrary rotation angle ϕ, i.e. find

$$\hat{R}_y(-\phi)\,\hat{L}_z\,\hat{R}_y(\phi).$$

4. A beam of particles (all in the same state) is subject to a simultaneous measurement of two observables: $\mathbf{L}^2$ and L_z. The measurement yields two pairs of values:
   $l=0$, $m=0$ with probability $3/4$,
   $l=1$, $m=-1$ with probability $1/4$.

   (a) Determine the state of the beam immediately before the measurement

   (b) If the particles in the beam with $l=1$, $m=-1$ are separated out and subjected to a measurement of L_x, what would be the possible outcomes and the corresponding probabilities of such a measurement?