PHYS 451: Quantum Mechanics I - Spring 2017
Homework #9, due Thursday April 13 in class

Addition of angular momenta, electron in magnetic field, identical particles

1. What are the Clebsch-Gordan coefficients involved in the expansion of the following states:

$$|2\,2\,1\,1\rangle, \quad |2\,1\,1\,1\rangle, \quad |2\,0\,1\,1\rangle, \quad |2\,-1\,1\,1\rangle, \quad |2\,-2\,1\,1\rangle \ ?$$

   Here $|l\,m\,l_1\,l_2\rangle$ stands for a state with a definite value of the total angular momentum (l) and its projection on the z-axis (m) formed by two particles that have orbital angular momenta l_1 and l_2.

   Hint: You might want to use the ladder operators, $\hat{L}_\pm = \hat{L}_{1\pm} + \hat{L}_{2\pm}$.

2. A neutral atom has a single valence electron that is bound in a state with orbital angular momentum quantum number $l = 1$.

   (a) What are possible eigenvalues of $\hat{L}^2$ and $\hat{L}_z$?

   (b) What is the value of the spin angular momentum quantum number, s? What are the possible eigenvalues of $\hat{S}^2$ and $\hat{S}_z$?

   (c) The magnetic moment for the neutral atom is $\boldsymbol{\mu} = -\frac{e}{2m_e}(\mathbf{L} + 2\mathbf{S})$. What are possible eigenvalues of $\hat{\mu}_z$ for this atom?

   (d) Suppose a beam of these atom is sent through a Stern-Gerlach apparatus. How many parallel beams will emerge?

3. Problem 4.33 in Griffiths.

4. Two non-interacting particles of mass m are in an infinite square well ($0 < x < a$) and occupy single particle eigenstates ψ_k and ψ_n respectively. Find the probability that both particles are *simultaneously* found in the left half of the well. Consider three cases:

   (a) The particles are distinguishable

   (b) The particles are identical bosons

   (c) The particles are identical fermions