

Before the sudden change in the spring constant occurred, the particle was in the ground state with

$$E_0 = \frac{1}{2}\hbar\omega \quad \Psi_0 = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\frac{\alpha x^2}{2}}$$

where $\alpha = \frac{m\omega}{\hbar}$

Right after the change occurred the initial wave function was still $\Psi_0(x)$ (because the potential was changed instantaneously). However, the eigenfunctions and the energy eigenvalues of the new Hamiltonian are different. They are

$$E'_0 = \frac{1}{2}\hbar\omega' = \hbar\omega \quad \Psi'_0 = \left(\frac{\alpha'}{\pi}\right)^{1/4} e^{-\frac{\alpha' x^2}{2}} = \left(\frac{2\alpha}{\pi}\right)^{1/4} e^{-\frac{\alpha x^2}{2}}$$

$$E'_1 = \frac{3}{2}\hbar\omega' = 3\hbar\omega \quad \Psi'_1 = A x e^{-\frac{\alpha x^2}{2}} \quad A = \text{const}$$

$$E'_2 = \frac{5}{2}\hbar\omega' = 5\hbar\omega \quad \Psi'_2 = \dots$$

$$E'_3 = \frac{7}{2}\hbar\omega' = 7\hbar\omega \quad \Psi'_3 = \dots$$

and so on

It is clear that there are no eigenstates with the energy $\frac{1}{2}\hbar\omega$, $\frac{3}{2}\hbar\omega$, $2\hbar\omega$, and $\frac{5}{2}\hbar\omega$. So

a) $P(E = \frac{1}{2}\hbar\omega) = 0$

e) $P(E = \frac{5}{2}\hbar\omega) = 0$

c) $P(E = \frac{3}{2}\hbar\omega) = 0$

d) $P(E = 2\hbar\omega) = 0$

- b) The initial wave function (after the change) can be expanded in terms of the stationary states of the new Hamiltonian, $\Psi'_n(x)$:

$$\Psi(x, t=0) = \Psi_0(x) = \sum_{n=0}^{\infty} c_n \Psi'_n(x)$$

$$c_0 = \int_{-\infty}^{+\infty} \Psi_0(x) \Psi'_0(x) dx = \left(\frac{\alpha}{\pi}\right)^{1/4} \left(\frac{2\alpha}{\pi}\right)^{1/4} \int_{-\infty}^{+\infty} e^{-\frac{\alpha x^2}{2}} e^{-\frac{\alpha x^2}{2}} dx = 2^{1/4} \sqrt{\frac{\alpha}{\pi}}.$$

$$\int_{-\infty}^{+\infty} e^{-\frac{3}{2}\alpha x^2} dx = 2^{1/4} \sqrt{\frac{\pi}{\alpha}} \sqrt{\frac{\pi}{\frac{3}{2}\alpha}} = \sqrt{\frac{2}{3}} 2^{1/4}$$

Then the probability of measuring $E = \hbar\omega$ is

$$P(E = \hbar\omega) = |c_0|^2 = \frac{2}{3}\sqrt{2} \approx 0.943$$