

a) The initial wave function can be written as

$$\psi(x,0) = \begin{cases} 2A f(x), & x > 0 \\ 0, & x \leq 0 \end{cases}$$

We also know that  $\int_{-\infty}^{+\infty} |f(x)|^2 dx = 1$ . Given the odd nature of  $f(x)$  this yields

$$\int_0^{\infty} |f(x)|^2 dx = \int_{-\infty}^0 |f(x)|^2 dx = 1/2$$

Then the normalization of  $\psi(x,0)$  gives

$$1 = \int_{-\infty}^{+\infty} |\psi(x,0)|^2 dx = 4A^2 \int_0^{\infty} |f(x)|^2 dx = 2A^2 \Rightarrow A = \frac{1}{\sqrt{2}}$$

b)  $\rho(x=0, t=0) = 0$  since  $f(x)$  is odd.

$$c) P(-\infty \leq x \leq 0) = \int_{-\infty}^0 |\psi(x,0)|^2 dx = 0$$

$$d) P(0 \leq x \leq +\infty) = \int_0^{\infty} |\psi(x,0)|^2 dx = 1$$

e) Neither the initial wavefunction  $\psi(x,0)$  nor the wave function at time  $t$  (i.e.  $\psi(x,t)$ ) have a definite parity.

f) The time-dependent wave function is

$$\psi(x,t) = \sum_{n=0}^{\infty} c_n \psi_n(x) e^{-\frac{iE_n t}{\hbar}} \quad c_n = \frac{1}{\sqrt{2}} \int_{-\infty}^0 f(x) \psi_n(x) dx$$

where  $E_n = \hbar\omega(n + \frac{1}{2})$  and  $\psi_n(x)$  are eigenfunctions of the harmonic oscillator. Up to a common phase factor  $e^{-\frac{i\omega t}{2}}$  this becomes

$$\psi(x,t) = \sum_{n=0}^{\infty} c_n \psi_n e^{-in\omega t} = \sum_{n=0}^{\infty} c_n \psi_n (e^{-i\omega t})^n$$

Note that  $\psi_n(x)$  themselves are either even functions ( $n=0, 2, 4, \dots$ ) or odd ones ( $n=1, 3, 5, \dots$ ).

When  $t = \frac{\pi}{\omega}(2k+1)$  where  $k$  is an integer we get

$$\psi(x,t) = \sum_n c_n \psi_n(x) (-1)^n = \sum_n c_n \psi_n(-x) = \psi(-x,0)$$

but

$$\Psi(-x, 0) = \begin{cases} 0 & , x \geq 0 \\ 2A f(-x) & , x < 0 \end{cases}$$

So the expression is zero for  $x > 0$  and the probability density  $|\Psi(-x, 0)|^2$  is nonzero only in the region  $x < 0$

f) Similarly to the previous case, consider the moment of time  $t = \frac{\pi}{\omega} 2k$  where  $k$  is any positive integer. Then  $e^{-i\omega t} = 1$

$$\Psi(x, t) = \sum_n c_n \Psi_n(x) \cdot 1^n = \Psi(x, 0)$$

Obviously  $|\Psi(x, 0)|^2$  can be greater than zero only for  $x > 0$ , which means that with probability equal to 1 the particle is in the region  $x > 0$