

① Here we use the following relation between the coupled and uncoupled representations of states of two subsystems with spins s_1 and s_2 :

$$|SM\rangle = \sum_{m_1+m_2=M} \langle s_1 m_1 s_2 m_2 | SM \rangle |s_1 m_1\rangle |s_2 m_2\rangle$$

In our case $S=3$ and $M=1$, so

$$|31\rangle = \sqrt{\frac{1}{15}} |22\rangle |1-1\rangle + \sqrt{\frac{8}{15}} |21\rangle |10\rangle + \sqrt{\frac{2}{5}} |20\rangle |11\rangle$$

Therefore, we might get the values $m_2=0, 1, 2$ with the corresponding probabilities

$$P(m_2=0) = \frac{2}{5} \quad P(m_2=1) = \frac{8}{15} \quad P(m_2=2) = \frac{1}{15}$$

② Here we use the relation

$$|\ell m_\ell\rangle |s m_s\rangle = \sum_J \langle JM | \ell m_\ell s m_s \rangle |JM\rangle$$

In our case $\ell=1$ $m_\ell=0$ $S=\frac{1}{2}$ $m_S=-\frac{1}{2}$, so

$$|10\rangle |\frac{1}{2}-\frac{1}{2}\rangle = \sqrt{\frac{2}{3}} |\frac{3}{2}-\frac{1}{2}\rangle + \sqrt{\frac{1}{3}} |\frac{1}{2}-\frac{1}{2}\rangle$$

and

$$P(J=\frac{3}{2}) = \frac{2}{3} \quad \hat{J}^2 = J(J+1)\hbar^2 = \frac{15}{2}\hbar^2$$

$$P(J=\frac{1}{2}) = \frac{1}{3} \quad \hat{J}^2 = J(J+1)\hbar^2 = \frac{3}{4}\hbar^2$$