PHYS 451 Quantum Mechanics I (Spring 2018)
Homework #2, due Thursday Feb 1 in class

Wave function, expectation values, Heisenberg uncertainty principle, particle in a box

1. Consider a particle of mass m in an infinite square well ($-b \leq x \leq b$; where b is a positive constant).

   (a) What are the energies, E_i, and eigenstates, ψ_i, of the particle?

   (b) Suppose the initial state of the particle is given by

   $$\Psi(x, t = 0) = \begin{cases} C(b - |x|), & -b \leq x \leq b \\ 0, & |x| \geq b \end{cases},$$

   where C is a constant. If a measurement of the energy is made at $t > 0$, what is the probability that the values E_1, E_2, and E_3 are obtained?

2. Consider the ground state wave function of the harmonic oscillator given by

   $$\psi_0(x) = Ce^{-\alpha x^2/2},$$

   where $\alpha = \frac{m\omega}{\hbar}$ (m is the mass and ω is the angular frequency of the oscillator).

   (a) Find the normalization constant, C.

   (b) Compute the expectation values of of the position, momentum, and their squares, i.e. $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, and $\langle p^2 \rangle$.

   (c) Verify that the Heisenberg uncertainty principle holds for this state.

3. A particle of mass m moves in the harmonic oscillator potential. Its initial state is given by

   $$\Psi(x, 0) = A[4\phi_0(x) - 3i\phi_1(x)],$$

   where ϕ_0 and ϕ_1 are the ground and first excited state wave functions of the oscillator.

   (a) Is $\Psi(x, t)$ is a stationary state? Explain why.

   (b) Determine the normalization constant A.

   (c) Write out $\rho(x, t) = |\Psi(x, t)|^2$. Make it clear that $\rho(x, t)$ is a nonnegative function.

   (d) Will the system ever return to its initial state, and if so, at what time?

   (e) Compute $\langle H \rangle$

   (f) Compute $\langle x \rangle$

   (g) Compute $\langle p \rangle$

4. A particle of mass m moves in the following potential:

   $$V(x) = \frac{kx^2}{2} - ax.$$

   Find the energies and eigenstates of the particle.