PHYS 451 Quantum Mechanics I (Spring 2018)
Homework #4, due Thursday Feb 15 in class

Transmission and reflection of a particle, Raising and lowering operators, Commutators

1. Find the transmission and reflection coefficients for the potential in the form of a step-function:
$$V(x) = \begin{cases} 0 & , x < 0 \\ V_0 & , x \geq 0 \end{cases},$$
where $V_0 > 0$. Assume that the particle is incident from the left and the energy of the particle is greater than V_0. Examine the limiting cases $E \to V_0$ and $E \to \infty$.

2. Using the formalism of the raising and lowering operators ($a^\dagger$ and a) compute the following general matrix elements in the basis of harmonic oscillator functions $\psi_n(x)$:

   (a) $\langle \psi_n | x | \psi_m \rangle$
   (b) $\langle \psi_n | x^2 | \psi_m \rangle$
   (c) $\langle \psi_n | p | \psi_m \rangle$
   (d) $\langle \psi_n | p^2 | \psi_m \rangle$

   Then find how the uncertainty principle holds for state n, i.e.

   (e) compute $\Delta x \Delta p$ for state ψ_n.

   Hint: first express x and p in terms of $a^\dagger$ and a, then recall from lecture how $a^\dagger$ and a act on the eigenfunctions of the Hamiltonian.

3. The orbital angular momentum operator is defined as
$$\hat{\mathbf{L}} = \hat{\mathbf{r}} \times \hat{\mathbf{p}} = \begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ \hat{x} & \hat{y} & \hat{z} \\ \hat{p}_x & \hat{p}_y & \hat{p}_z \end{vmatrix},$$
where $|...|$ stands for the determinant and $\mathbf{e}_i$ are unit vectors. The kinetic energy operator is given by:
$$\hat{T} = \frac{\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2}{2m} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right).$$
Find the following commutators:

   (a) $[\hat{x}, \hat{p}_x]$
   (b) $[\hat{x}, \hat{p}_y]$
   (c) $[\hat{y}, \hat{p}_x]$
   (d) $[\hat{p}_x, \hat{p}_y]$
   (e) $[\hat{x}, \hat{T}]$
   (f) $[\hat{p}_x, \hat{T}]$
   (g) $[\hat{x}, \hat{L}_x]$
   (h) $[\hat{x}, \hat{L}_y]$
   (i) $[\hat{x}, \hat{L}_x^2]$
   (j) $[\hat{x}, \hat{L}_y^2]$
   (k) $[\hat{L}_x, \hat{L}_x]$
   (l) $[\hat{L}_x, \hat{L}_y]$