Transmission and reflection of a particle, Raising and lowering operators, Commutators

1. Find the transmission and reflection coefficients for the potential in the form of a step-function:

\[ V(x) = \begin{cases} 
0, & x < 0 \\
V_0, & x \geq 0 
\end{cases} \]

where \( V_0 > 0 \). Assume that the particle is incident from the left and the energy of the particle is greater than \( V_0 \). Examine the limiting cases \( E \to V_0 \) and \( E \to \infty \).

2. Using the formalism of the raising and lowering operators \((a^\dagger, a)\) compute the following general matrix elements in the basis of harmonic oscillator functions \( \psi_n(x) \):

(a) \( \langle \psi_n | x | \psi_m \rangle \)
(b) \( \langle \psi_n | x^2 | \psi_m \rangle \)
(c) \( \langle \psi_n | p | \psi_m \rangle \)
(d) \( \langle \psi_n | p^2 | \psi_m \rangle \)

Then find how the uncertainty principle holds for state \( n \), i.e.

(e) compute \( \Delta x \Delta p \) for state \( \psi_n \).

*Hint: first express \( x \) and \( p \) in terms of \( a^\dagger \) and \( a \), then recall from lecture how \( a^\dagger \) and \( a \) act on the eigenfunctions of the Hamiltonian.*

3. The orbital angular momentum operator is defined as

\[ \hat{\mathbf{L}} = \hat{\mathbf{r}} \times \hat{\mathbf{p}} = \begin{vmatrix} 
e_x & \ne_y & \ne_z \\
\hat{x} & \hat{y} & \hat{z} \\
\hat{p}_x & \hat{p}_y & \hat{p}_z \end{vmatrix}, \]

where \(|...|\) stands for the determinant and \( \ne_x \) are unit vectors. The kinetic energy operator is given by:

\[ \hat{T} = \frac{\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2}{2m} = -\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right). \]

Find the following commutators:

(a) \([\hat{x}, \hat{p}_x]\)
(b) \([\hat{x}, \hat{p}_y]\)
(c) \([\hat{y}, \hat{p}_x]\)
(d) \([\hat{p}_x, \hat{p}_y]\)
(e) \([\hat{x}, \hat{T}]\)
(f) \([\hat{p}_x, \hat{T}]\)
(g) \([\hat{x}, \hat{L}_x]\)
(h) \([\hat{x}, \hat{L}_y]\)
(i) \([\hat{x}, \hat{L}_z^2]\)
(j) \([\hat{x}, \hat{L}_y^2]\)
(k) \([\hat{L}_x, \hat{L}_x]\)
(l) \([\hat{L}_x, \hat{L}_y]\)

Found an error or need a clarification? Email the instructor at sergiy.bubin@nu.edu.kz