

**PHYS 451 Quantum Mechanics I (Spring 2018)**  
**Homework #6, due Thursday March 1 in class**

General formalism of quantum mechanics, Spherical harmonics

1. Problem 3.22 in Griffiths.
2. Problem 3.23 in Griffiths.
3. By considering the rate of change of operator  $xp_x$  derive the virial theorem in 1D:

$$2\langle T \rangle = \left\langle x \frac{\partial V}{\partial x} \right\rangle,$$

where  $T$  and  $V$  are the kinetic and potential energy respectively. Next, generalize the theorem to the 3D case, i.e. consider operator  $\mathbf{r} \cdot \mathbf{p}$ .

4. An electron is confined in an infinitely deep cubic potential well, whose sides are of length  $a$  and are parallel to the  $x$ ,  $y$ , and  $z$ -axes.
  - (a) Write the time-independent wave function corresponding to the states of the lowest and second lowest energy.
  - (b) What is the degeneracy of energy levels for this system?
  - (c) Find the number of states,  $N$ , that have energy less than some given  $E$ .
5. Write all spherical harmonics up to  $l = 2$  (there are nine of them) in Cartesian form, i.e. give expressions in terms of  $x$ ,  $y$ ,  $z$ , and  $r = \sqrt{x^2 + y^2 + z^2}$ . You can either use the Rodrigues formula for the Legendre polynomials or start with the given expressions for  $Y_l^m$  in terms of  $\theta$  and  $\phi$ . In any event you must show your work.