PHYS 451 Quantum Mechanics I (Spring 2018)
Homework #7, due Saturday March 10 in class

Motion in 3D, central potentials

1. Consider a particle of mass m that is constrained to move in between two concentric impenetrable spheres of radius a and b. In other words, the particle moves in the following central potential:

$$V(r) = \begin{cases} 0, & a < r < b \\ \infty, & \text{otherwise} \end{cases}$$

   Find the ground state energy and wave function.

2. Problem 4.13 in Griffiths.

3. Consider a hydrogen atom. Its initial state is given by the wave function

$$\Psi(\mathbf{r}, t = 0) = \frac{1}{\sqrt{10}}\Big(2\psi_{100}(\mathbf{r}) + \psi_{210}(\mathbf{r}) + \sqrt{2}\psi_{211}(\mathbf{r}) + \sqrt{3}\psi_{21-1}(\mathbf{r})\Big),$$

   where the subscripts are the values of the quantum numbers n, l, and m.

   (a) Find the expectation value of the energy

   (b) Find the probability (as a function of time) that a measurement of $\mathbf{L}^2$ and L_z yields $2\hbar^2$ and $+\hbar$ respectively.

   (c) What is the probability of finding the electron within 1 Å of the proton at time $t = 0$? You can make some reasonable approximations here if you want to.

   (d) What is $\Psi(\mathbf{r}, t)$?

4. Problem 4.38 in Griffiths.