PHYS 451 Quantum Mechanics I (Spring 2018)
Homework #8, due Tuesday March 27 in class

Angular momentum

1. State $|\psi\rangle$ is an eigenstate of $\hat{\mathbf{L}}^2$ and $\hat{L}_z$, i.e.

$$\hat{\mathbf{L}}^2|\psi\rangle = \hbar^2 l(l+1)|\psi\rangle \quad \text{and} \quad \hat{L}_z|\psi\rangle = \hbar m|\psi\rangle$$

   Find $\langle\hat{L}_x\rangle$ and $\langle\hat{L}_x^2\rangle$ in this state.
   Hint: it may be helpful to take into account the symmetry with respect to x and y.

2. A spinless particle has the following wave function:

$$\psi = A(x+y+2z)e^{-\beta r},$$

   where A and β are positive constants and $r = \sqrt{x^2+y^2+z^2}$.

   (a) What is the total angular momentum of the particle?

   (b) What is the expectation value of the z-component of the angular momentum?

   (c) What are the probabilities of getting $+2\hbar$ and $+\hbar$ and 0 upon measuring the z-component of the angular momentum?

   (d) What is the probability of finding the particle at angles θ and ϕ (azimuthal and polar angle respectively) in solid angle $d\Omega$?

3. A beam of particles (all in the same state) is subject to a simultaneous measurement of two observables: $\mathbf{L}^2$ and L_z. The measurement yields two pairs of values:
   $l = 0$, $m = 0$ with probability 3/4,
   $l = 1$, $m = -1$ with probability 1/4.

   (a) Determine the wave function of the particles in beam immediately before the measurement

   (b) If the particles in the beam with $l = 1$, $m = -1$ are separated out and subjected to a measurement of L_x, what would be the possible outcomes and the corresponding probabilities of such a measurement?

4. Problem 4.25 in Griffiths.