PHYS 451 Quantum Mechanics I (Spring 2018)
Homework #9, due Tuesday April 10 in class

Matrix representation of operators, Addition of angular momenta, Stern-Gerlach effect

1. *This is the problem that we started but did not finish in the recitation on April 2nd.* Consider a three-level quantum system. In the absence of interaction between the levels, they are all degenerate with the energy equal to ε. Let us denote the states as $|1\rangle$, $|2\rangle$, and $|3\rangle$. Initially, at $t = 0$ the system is in state $|1\rangle$.

   (a) What is the time-dependent wave function at $t > 0$ when there is no interaction? What is the probability of transition to states $|2\rangle$ and $|3\rangle$?

   (b) Now we turn on the interaction $\hat{V}$. The interaction is such that in the basis of states $|1\rangle$, $|2\rangle$, and $|3\rangle$ its matrix elements are

   $$V_{ij} = \begin{cases} 0, & i = j \\ \alpha, & i \neq j \end{cases}$$

   where α is some real constant. What is the time-dependent wave function at $t > 0$ in this case? What is the probability of transition to states $|2\rangle$ and $|3\rangle$?

2. What are the Clebsch-Gordan coefficients involved in the expansion of the following states:

   $$|2\,2\,1\,1\rangle, \quad |2\,1\,1\,1\rangle, \quad |2\,0\,1\,1\rangle, \quad |2\,-1\,1\,1\rangle, \quad |2\,-2\,1\,1\rangle\ ?$$

   Here $|l\,m\,l_1\,l_2\rangle$ stands for a state with a definite value of the total angular momentum (l) and its projection on the z-axis (m) formed by two particles that have orbital angular momenta l_1 and l_2.

   Hint: You might want to use the ladder operators, $\hat{L}_\pm = \hat{L}_{1\pm} + \hat{L}_{2\pm}$.

3. (a) For a particle with spin 1/2, find two spin functions, $|\uparrow_{\mathbf{n}}\rangle$ and $|\downarrow_{\mathbf{n}}\rangle$, that correspond to a definite projection ($+\hbar/2$ or $-\hbar/2$ respectively) of the spin on an arbitrary axis defined by unit vector $\mathbf{n}$. Express these functions in terms of the eigenstates of $\hat{S}_z$ operator. *Hint: it is convenient to consider operator* $\mathbf{n}\cdot\boldsymbol{\sigma} = n_x\sigma_x + n_y\sigma_y + n_z\sigma_z$.

   (b) Now, let us assume that the particle is initially prepared in a state that corresponds to the negative projection of the spin on the z-axis. If the projection of the spin on axis $\mathbf{n}$ is measured, what will be the probability of getting the value $+\hbar/2$?

4. A neutral atom has a single valence electron that is bound in a state with orbital angular momentum quantum number $l = 1$.

   (a) What are possible eigenvalues of $\hat{L}^2$ and $\hat{L}_z$?

   (b) What is the value of the spin angular momentum quantum number, s? What are the possible eigenvalues of $\hat{S}^2$ and $\hat{S}_z$?

   (c) The total magnetic moment for the atom is $\boldsymbol{\mu} = -\frac{e}{2m_e}(\mathbf{L} + 2\mathbf{S})$. What are possible eigenvalues of $\hat{\mu}_z$ for this atom?

   (d) Suppose a beam of these atom is sent through a Stern-Gerlach apparatus. How many parallel beams will emerge?