PHYS 451 Quantum Mechanics I (Spring 2020)
Instructor: Sergiy Bubin
Online Final Exam

General instructions (please read carefully)

- All problems are worth the same number of points (although some might be more difficult than the others).
- No communication related to this exam is allowed with classmates or other individuals, regardless of the type and nature of such communication. The only individual you can communicate with is the instructor.
- You can use lecture notes or textbooks as you wish.
- You can also use calculators, computers, and computer algebra software packages if you find it helpful.
- Show all your work, explain your reasoning. Answers without explanations will receive no credit (not even partial one).

Submission guidelines (please read carefully)

- Write legibly. If I cannot read and understand it then I will not be able to grade it.
- Do not use red pen.
- Make sure pages are ordered properly. It is recommended to start each problem on a new page.
- Use plain white paper (not ruled or checkered paper).
- The file submitted must be in the PDF format.
- If possible use a scanner, set the resolution to 300 dpi or higher.
- If a scanner is not available, take high-resolution photos with your phone, but make sure the lighting is adequate. Also, make sure pages lie flat on the surface so that everything is in good focus. Pages should be rotated properly and appear in portrait mode.
- Make sure there are no multiple versions/copies/drafts of the same problem as it may be confusing.
- Look through your file before you submit it and make sure everything is ok with it, e.g. all fine details such as small indicies, superscripts, etc are clearly readable, no pages are missing, pages are ordered and have proper orientation, etc.
- Submit your file via Google Classroom. Do not wait until the last minute as technical issues or intermittent internet connection may result in a late submission.

Problem 1. Consider a 1D quantum harmonic oscillator of frequency ω, initially prepared in the state $|\psi\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)$, where $|n\rangle$ stands for an egenstate corresponding to quantum number n.

(a) Is this harmonic oscillator in a stationary state?

(b) Will the Heisenberg uncertainty principle hold at any later moment of time?

Do not just answer questions. Make sure to prove, show, or explain your point.

Problem 2. Some physical system has the observables that are represented by the following operators:

$$A = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{pmatrix}, \qquad C = \begin{pmatrix} 0 & 3 & 0 \\ 3 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}, \qquad D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}.$$

(a) What are the possible results of the measurements of these observables?

(b) Which of these observables are mutually compatible? Find a basis of common eigenstates.

(c) Find the operator that, when acting on an arbitrary state, leaves only the component corresponding to the positive values of observable D.

Problem 3.

(a) Find the eigenstates of the position operator, $\hat{x}$, in the coordinate representation.

(b) Now consider two particles in 1D that are stuck to each other and move with a definite value of the total linear momentum P $(= K\hbar)$. Write the wave function of this system in the coordinate respresentation. You can ignore the normalization.

(c) What is the wave function of the system in the momentum representation?

(d) Suppose we measure the coordinate of the first particle and obtain a. In what state will the second particle be left (i.e. what state will the second particle be projected onto)?

(e) Suppose we measure the momentum of the first particle and obtain p_0 $(= k_0\hbar)$. In what state will the second particle be left?

Problem 4. Consider two spin 1/2 particles. Suppose the operator that describes the interation between them has the following form

$$V = a + \beta\, \mathbf{S}_1 \cdot \boldsymbol{S}_2 = a + b\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2,$$

where a, and β are real constants, $b = \beta\hbar^2/4$, and σ's are Pauli spin matrices. The total spin of the system is $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$.

(a) Can V, $\mathbf{S}^2$, and S_z be measured simultaneously?

(b) Determine the matrix form of V in the uncoupled representation.

(c) Determine the matrix form of V in the coupled representation.

Make sure to indicate clearly in which order you place basis states as the matrix form of V depends on this order.