

**PHYS 451 Quantum Mechanics I (Spring 2020)**  
**Homework #1, due Tuesday January 28 in class**

Review of basic probability theory.

1. Problem 1.1 in Griffiths
2. Consider the following wave function:

$$\psi(x, t) = A x e^{-\beta|x|+i\lambda t}$$

where  $\beta$  and  $\lambda$  are some real constants and  $\beta > 0$ .

- (a) Determine the normalization factor,  $A$ .
  - (b) Compute the expectation values  $\langle x \rangle$  and  $\langle x^2 \rangle$ .
  - (c) Find  $\sigma$ , the standard deviation of  $x$ .
  - (d) Sketch the graph of  $|\psi|^2$  as a function of  $x$ , and mark the points  $\langle x \rangle + \sigma$  and  $\langle x \rangle - \sigma$ , to illustrate how  $\sigma$  represents the “spread” of the distribution in  $x$ . What is the probability that the particle is found outside of this range?
3. Some quantity  $k$  may take integer values in the range  $1 \dots K$ , where  $K$  is a positive integer. The probability of each of those values is known to be inversely proportional to  $k$ .
    - (a) Find probability  $P(k)$
    - (b) Find the average value  $\langle k \rangle$ ?
    - (c) Find the variance  $\langle k^2 \rangle$ ?
    - (d) Calculate the uncertainty  $\Delta k$ ?
  4. Suppose we choose at random a real number  $x$  from the interval  $[2, 10]$ .
    - (a) Find the probability density function  $\rho(x)$
    - (b) Find the probability of getting a number that lies in subinterval  $[a, b]$ .
    - (c) Find the probability that  $x^2 - 12x + 35$  is positive.