PHYS 451 Quantum Mechanics I (Spring 2020)
Homework #6, due Tuesday Apr 14 at 11:59pm

Spherical harmonics, motion in central potential

1. Write all spherical harmonics up to $l = 2$ (there are nine of them) in Cartesian form, i.e. give expressions in terms of x, y, z, and $r = \sqrt{x^2 + y^2 + z^2}$. You can either use the Rodrigues formula for the Legendre polynomials or start with the given expressions for Y_l^m in terms of θ and ϕ. In any event you must show your work.

2. Consider a particle of mass m that is constrained to move in between two concentric impenetrable spheres of radius a and b. In other words, the particle moves in the following central potential:

$$V(r) = \begin{cases} 0, & a < r < b \\ \infty, & \text{otherwise} \end{cases}$$

   Find the ground state energy and wave function.

3. Consider a hydrogen atom. Its initial state is given by the wave function

$$\Psi(\mathbf{r}, t\!=\!0) \;=\; A\Big(3\psi_{100}(\mathbf{r}) - i\psi_{211}(\mathbf{r}) - 2\psi_{210}(\mathbf{r}) + \psi_{21-1}(\mathbf{r}) + i\psi_{321}(\mathbf{r})\Big),$$

   where A is a normalization constant and subscripts (e.g. in ψ_{100}) stand for the quantum numbers n, l, and m.

   (a) Find the normalization constant A.

   (b) Find the expectation value of the energy.

   (c) Find the probability (as a function of time) that a measurement of $\mathbf{L}^2$ and L_z yields $2\hbar^2$ and $+\hbar$ respectively.

   (d) What is the probability of finding the electron within 1×10^{-12} m of the proton at time $t = 0$? You can make some reasonable approximations here if you want to.

   (e) What is $\Psi(\mathbf{r}, t)$?

4. Problem 4.38 in Griffiths.