PHYS 451 Quantum Mechanics I (Spring 2020)
Homework #8, due Monday Apr 27 at 11:59pm

Angular momentum, Addition of angular momenta, Angular momentum in magnetic field, Identical particles, Atoms

1. An electron is in the spin state $\chi = A \begin{pmatrix} 3i \\ 4 \end{pmatrix}$.
   (a) Determine the normalization constant A.
   (b) Find the expectation values of S_x, S_y, and S_z.
   (c) Find the uncertainties ΔS_x, ΔS_y, and ΔS_z.
   (d) Confirm that your results for different components of spin are consistent with the uncertainty principle.

2. Consider two particles with spin equal to 1 and 2. The interaction beweeen the particles is described by the Hamiltonian $\hat{H} = \beta\, \mathbf{S}_1 \cdot \mathbf{S}_2$ ($\beta > 0$). Find the energy levels and their degeneracies.

3. Consider a beam of particles, all in the same state. The particles are subject to a simultaneous measurement of two observables: $\mathbf{L}^2$ and L_z. The measurement gives two outcomes:
   $l = 0$, $m = 0$ with probability 3/4,
   $l = 1$, $m = -1$ with probability 1/4.
   (a) Determine the state of the beam before the measurement
   (b) If the particles in the beam with $l = 1$, $m = -1$ are separated out and subjected to a measurement of L_x, determine possible outcomes and the corresponding probabilities in such a measurement.

4. A neutral atom has a single valence electron that is bound in a state with orbital angular momentum quantum number $l = 1$.
   (a) What are possible eigenvalues of $\hat{L}^2$ and $\hat{L}_z$?
   (b) What is the value of the spin angular momentum quantum number, s? What are the possible eigenvalues of $\hat{S}^2$ and $\hat{S}_z$?
   (c) The magnetic moment for the neutral atom is $\boldsymbol{\mu} = -\frac{e}{2m_e}(\mathbf{L} + 2\mathbf{S})$. What are possible eigenvalues of $\hat{\mu}_z$ for this atom?
   (d) Suppose a beam of these atom is sent through a Stern-Gerlach apparatus. How many parallel beams will emerge?

5. Two noninteracting particles of mass m are placed in 1D harmonic oscillator potential. One particle is in the ground state, while the other one is in the first excited state. Calculate the mean squared distance between the particles assuming that they are in the same spin state and are
   (a) distinguishable particles
   (b) identical bosons
   (c) identical fermions

6. Consider the oxygen atom. Write the electron configuration for the ground state. Ignoring the interaction between electrons, estimate the ground state energy and give this value either both in hartrees and electronvolts. Determine the ground state term symbol, explain your reasoning.

Found an error or need a clarification? Email the instructor at sergiy.bubin@nu.edu.kz