PHYS 451 Quantum Mechanics I (Spring 2020)
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Online Midterm Exam 2

Instructions:

- All problems are worth the same number of points (although some might be more difficult than the others).
- No communication related to this exam is allowed with classmates or other individuals, regardless of the type and nature of such communication. The only individual you can communicate with is the instructor.
- You can use lecture notes or textbooks as you wish.
- You can also use calculators, computers, and computer algebra software packages if you find it helpful.
- Show all your work, explain your reasoning. Answers without explanations will receive no credit (not even partial one).
- Write legibly. If I cannot read and understand it then I will not be able to grade it.
- Make sure pages are ordered properly. It is recommended to start each problem on a new page.
- The file submitted must be in the PDF format.
- Make sure there are no multiple versions/copies/drafts of the same problem as it may be confusing.

Problem 1. Positronium is a hydrogen-like atom in which the proton (nucleus) is replaced with a positron. While this system is unstable against electron–positron annihilation, it lives long enough to detect it and study its properties.

(a) What is the wavelength of the transition between the ground and first excited state (analogue of the Lyman-alpha line in hydrogen) in positronium? Give your answer in nanometers.

(b) What is the most probable distance between the positron and electron in the ground state of positronium? Give your answer in meters or Angstroms.

(c) The annihilation rate in positronium, Γ, ($\Gamma = 1/\tau$, where τ is the lifetime) is proportional to the probability that the positron and electron happen to be in (nearly) the same point in space. Suppose we know the lifetime of positronium in the ground state, τ_0. What is the lifetime of positronium in the first excited state then? Note that the first excited state is degenerate and each state from that degenerate multiplet may have a different lifetime. Ignore the fact that the particles have spin.

Problem 2. The total wave function, which depends on both spatial (r, θ, ϕ) and spin variables, of an electron in the hydrogen atom is given by the following expression:

$$\Psi = A \begin{pmatrix} \sin\phi\,\sin\theta\,\frac{r}{a_0}\exp\left[-\frac{r}{2a_0}\right] \\ 2\exp\left[-\frac{r}{a_0}\right] \end{pmatrix},$$

Here A is a normalization constant and a_0 is the Bohr radius. What values and with what probabilities will be obtained if we measure

(a) Energy

(b) Square of the orbital angular momentum

(c) Projection of the orbital angular momentum on the z-axis

(d) Projection of the spin on the z-axis

(e) Projection of the total angular momentum (orbital ang. mom. + spin) on the z-axis

Problem 3. Consider an electron in the state corresponding to the positive projection of its spin on the z axis. Now we measure the projection of the spin on axis z', which makes angle θ with z. What are the probabilities of getting the positive and negative values?