

PHYS 451 Quantum Mechanics I (Spring 2020)
Quiz #3

Find the following commutators:

(a) $[\hat{x}, \hat{p}_x^2]$

(b) $[\hat{p}_x \hat{x}, \hat{x} \hat{p}_x]$

Solution:

First, let us recall the commutator

$$[\hat{p}, \hat{x}] = -i\hbar \left[\frac{d}{dx}, x \right] = -i\hbar$$

which can be recast as

$$\hat{p}\hat{x} = \hat{x}\hat{p} - i\hbar \quad \text{or} \quad \hat{x}\hat{p} = \hat{p}\hat{x} + i\hbar$$

Then

(a) $[\hat{x}, \hat{p}^2] = \hat{x}\hat{p}\hat{p} - \hat{p}\hat{p}\hat{x} = (\hat{p}\hat{x} + i\hbar)\hat{p} - \hat{p}(\hat{x}\hat{p} - i\hbar) = 2i\hbar\hat{p}$

(b) $[\hat{p}\hat{x}, \hat{x}\hat{p}] = \hat{p}\hat{x}\hat{x}\hat{p} - \hat{x}\hat{p}\hat{p}\hat{x} = (\hat{x}\hat{p} - i\hbar)\hat{x}\hat{p} - \hat{x}\hat{p}(\hat{x}\hat{p} - i\hbar) = 0$