What is the average magnitude of the electric field that the electron experiences due to its interaction with the proton in the ground state of the hydrogen atom? Give a symbolic answer and calculate the corresponding numerical value (in V/m or V/Å).

Solution:

The magnitude of the electric field due to the proton is given by

$$\mathcal{E} = \frac{1}{4\pi\epsilon_0}\frac{e}{r^2},$$

where r is the distance between the electron and proton and e is the elementary charge. The ground state wave function is

$$\psi = \frac{1}{\pi^{1/2}a_0^{3/2}}\,e^{-\frac{r}{a_0}},$$

where a_0 is the Bohr radius. The average electric field is then

$$\langle\mathcal{E}\rangle = \frac{1}{\pi a_0^3}\frac{e}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty}\frac{1}{r^2}\,e^{-\frac{2r}{a_0}}\,r^2\sin\theta\,d\phi\,d\theta\,dr = \frac{e}{\pi\epsilon_0 a_0^3}\int_0^{\infty}e^{-\frac{2r}{a_0}}\,dr = \frac{e}{2\pi\epsilon_0 a_0^2}.$$

The numerical value of which is

$$\langle\mathcal{E}\rangle \approx 1.03\times10^{12}\,\frac{\text{V}}{\text{m}} = 103\,\frac{\text{V}}{\text{Å}}.$$