PHYS 452: Quantum Mechanics II (Spring 2015)
Homework #2, due Thursday February 5, in class

1. A particle in the infinite square well of width a is subjected to a weak perturbation in the form
$$V'(x) = \begin{cases} \gamma x, & 0 < x < a \\ 0, & \text{otherwise} \end{cases}$$
where γ is a constant.

   (a) Find the first order correction to the energy levels.

   (b) What happens to the first-order energy correction in the limit of large n (n is the quantum number) for an arbitrary functional form of the perturbing potential, $V'(x)$.

2. Consider a rigid rotor constrained to rotate in the xy plane. The Hamiltonian of this system contains only the kinetic energy term:
$$H_0 = -\frac{\hbar^2}{2I}\frac{d^2}{d\phi^2},$$
where I is the moment of inertia of the rotor and ϕ is the rotation angle.

   (a) Find the eigenvalues and eigenfunctions of H_0.

   (b) Now let us assume the rotor has a fixed dipole moment, $\mathbf{p}$, in the xy plane. A weak electric field, $\mathbf{E}$, is applied. Find the changes in the energy levels to the first and second order in the field.

3. Using the perturbation theory consider the effect of the finite size of the nucleus in the hydrogen-like atom. Regard the nucleus of charge Z as a small uniformly charged sphere of radius $R_0 \ll a_0$, where a_0 is the Bohr radius.

   (a) Determine the electrostatic potential $V(r)$ between the nucleus and the electron as a function of r. If $V_0(r) = -\frac{Ze^2}{4\pi\varepsilon_0 r}$ is the potential due to a point charge, define $V' = V(r) - V_0(r)$.

   (b) What is the zero-order wave function (i.e. the wave function corresponding to the unperturbed potential V_0)?

   (c) Compute the first-order correction to the ground state energy.

4. A spin-1/2 particle moves in three-dimensional harmonic oscillator potential with frequency ω. It is also subject to (rather weak) spin interaction, so that the total Hamiltonian is
$$H = H_0 + H',$$
where
$$H_0 = -\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega^2 r^2,$$
$$H' = \lambda \mathbf{r}\cdot\boldsymbol{\sigma}.$$
In the above expression λ is a small constant and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are Pauli matrices. Compute the shift of the ground state energy due to the spin interaction to order $\mathcal{O}(\lambda^2)$.

Found an error or need a clarification? Email the instructor at sergiy.bubin@nu.edu.kz