Name: ____________________

PHYS 452: Quantum Mechanics II, Quiz #2

Instruction: use additional sheets if you find it necessary

Consider a quantum system with Hamiltonian H^0 that has just three energy levels, which are (in some units):

$$E_1^{(0)} = 1, \quad E_2^{(0)} = 1, \quad E_3^{(0)} = 2.$$

The system is subjected to a weak perturbation H'. The matrix elements of the perturbation Hamiltonian in the basis of the eigenstates of H^0 are:

$$H'_{ij} = a, \quad i,j = 1,2,3 \qquad |a| \ll 1.$$

Using the perturbation theory find corrections to the energy levels up to the second order.

Appendix: Perturbation theory formulae

$H = H^0 + \lambda H', \qquad E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \ldots, \qquad \psi_n = \psi_n^{(0)} + \lambda \psi_n^{(1)} + \lambda^2 \psi_n^{(2)} + \ldots$

$$E_n^{(1)} = H'_{nn}$$

$$\psi_n^{(1)} = \sum_m c_{nm} \psi_m^{(0)}, \quad c_{nm} = \begin{cases} \frac{H'_{mn}}{E_n^{(0)} - E_m^{(0)}}, & n \neq m \\ 0, & n = m \end{cases}$$

$$E_n^{(2)} = \sum_{m \neq n} \frac{|H'_{mn}|^2}{E_n^{(0)} - E_m^{(0)}}$$

$$\psi_n^{(2)} = \sum_m d_{nm} \psi_m^{(0)}, \quad d_{nm} = \begin{cases} \frac{1}{E_n^{(0)} - E_m^{(0)}} \left(\sum_{k \neq n} \frac{H'_{mk} H'_{kn}}{E_n^{(0)} - E_k^{(0)}} \right) - \frac{H'_{nn} H'_{mn}}{\left(E_n^{(0)} - E_m^{(0)}\right)^2}, & n \neq m \\ 0, & n = m \end{cases}$$