Name: ______________________

PHYS 452: Quantum Mechanics II, Quiz #3

Instruction: use additional sheets if you find it necessary

Consider a particle of mass m in the ground state ψ_1 of the 1D infinite square well ($0 \le x \le a$). At time $t = 0$ the potential is changed to

$$V(x) = \begin{cases} V_0, & 0 \le x \le \frac{a}{2} \\ 0, & \frac{a}{2} < x \le a \\ \infty, & \text{otherwise} \end{cases},$$

and at time $t = T$ the potential is reverted back to the original 1D infinite square well. Find the probability that the particle makes a transition to the first excited state ψ_2.

Appendix:

Infinite square well ($0 \le x \le a$)
Energy levels: $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$, $n = 1, 2, ..., \infty$
Eigenfunctions: $\phi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) \quad (0 \le x \le a)$
Matrix elements of the position: $\int\limits_0^a \phi_n^*(x) x\, \phi_k(x) dx = \begin{cases} a/2, & n = k \\ 0, & n \ne k;\ n \pm k \text{ is even} \\ -\frac{8nka}{\pi^2(n^2-k^2)^2}, & n \ne k;\ n \pm k \text{ is odd} \end{cases}$

Time-dependence of the wave function

$H(\mathbf{r}, t) = H^0(\mathbf{r}) + \lambda H'(\mathbf{r}, t), \qquad H^0\varphi_n = E_n^{(0)}\varphi_n, \qquad \psi(\mathbf{r}, t) = \sum\limits_n c_n(t)\varphi_n(\mathbf{r})e^{\frac{-iE_n^{(0)}t}{\hbar}},$

$i\hbar\frac{dc_n(t)}{dt} = \lambda \sum\limits_k H'_{nk} e^{i\omega_{nk}t} c_k(t), \qquad H'_{nk} = \langle\phi_n|H'|\phi_k\rangle, \qquad \omega_{nk} = \frac{E_n^{(0)} - E_k^{(0)}}{\hbar}$

Time-dependent perturbation theory formulae
If $c_n(t_0) = \delta_{nm}$ (e.g. $\psi(\mathbf{r}, t_0) = \varphi_m(\mathbf{r})$) and $\lambda H'$ is small then at time $t > t_0$
$c_n(t) = c_n^{(0)} + \lambda c_n^{(1)} + \lambda^2 c_n^{(2)} + \ldots$
where
$c_n^{(0)} = \delta_{nm}, \quad c_n^{(1)}(t) = \frac{1}{i\hbar}\int\limits_{t_0}^{t} H'_{nm}(t')\, e^{i\omega_{nm}t'} dt',$

$c_n^{(2)}(t) = \left(\frac{1}{i\hbar}\right)^2 \sum\limits_k \int\limits_{t_0}^{t} dt' \int\limits_{t_0}^{t'} H'_{nk}(t') H'_{km}(t'')\, e^{i\omega_{nk}t'} e^{i\omega_{km}t''} dt'', \qquad \ldots$

Fermi's golden rule
Transition rate: $\Gamma_{i\to f} = \frac{2\pi}{\hbar}|H'_{fi}|^2\, g(E_f)$, Probability: $P_{i\to f}(t) = \frac{2\pi t}{\hbar}|H'_{fi}|^2\, g(E_f)$

Useful trigonometric identities

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)] \qquad \cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$
$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)] \qquad \cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$