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PHYS 452: Quantum Mechanics II – Fall 2016
Quiz #3

A two-level quantum system with energy levels $E_1^{(0)} = \alpha$ and $E_2^{(0)} = \alpha$ is subject to a perturbative interaction H'. The matrix elements of the perturbation Hamiltonian in the basis of the eigenstates of the unperturbed Hamiltonian are $H'_{11} = \beta$, $H'_{12} = \beta$, $H'_{22} = \beta$ and it is known that $|\beta| \ll |\alpha|$. Using the perturbation theory find corrections to the energy levels up to the second order in H'.

Appendix: Perturbation theory formulae (from lecture)

$H = H^0 + \lambda H', \qquad E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \ldots, \qquad \psi_n = \psi_n^{(0)} + \lambda\psi_n^{(1)} + \lambda^2\psi_n^{(2)} + \ldots$

$$E_n^{(1)} = H'_{nn}$$

$$\psi_n^{(1)} = \sum_m c_{nm}\psi_m^{(0)}, \quad c_{nm} = \begin{cases} \frac{H'_{mn}}{E_n^{(0)} - E_m^{(0)}}, & n \neq m \\ 0, & n = m \end{cases}$$

$$E_n^{(2)} = \sum_{m \neq n} \frac{|H'_{mn}|^2}{E_n^{(0)} - E_m^{(0)}}$$

$$\psi_n^{(2)} = \sum_m d_{nm}\psi_m^{(0)}, \quad d_{nm} = \begin{cases} \frac{1}{E_n^{(0)} - E_m^{(0)}} \left(\sum_{k \neq n} \frac{H'_{mk}H'_{kn}}{E_n^{(0)} - E_k^{(0)}} \right) - \frac{H'_{nn}H'_{mn}}{\left(E_n^{(0)} - E_m^{(0)}\right)^2}, & n \neq m \\ 0, & n = m \end{cases}$$