PHYS 451 Quantum Mechanics II (Fall 2017)
Quiz #3

(a) A spin 1/2 particle with gyromagnetic ratio γ (the gyromagnetic ratio is the ratio of the particle's magnetic moment to its angular momentum) is placed in a magnetic field. The field lies in the yz-plane and has the form $\mathbf{B}_0 = (0, 4B, 3B)$. What are the possible energy levels of the particle? What are the corresponding states?

(b) Now an additional field is introduced. This weak field lies along the x-axis, $\mathbf{B}' = (\beta B, 0, 0)$, and its magnitude is much smaller than that of $\mathbf{B}_0$. In other words, $|\mathbf{B}'|/|\mathbf{B}_0| \ll 1$ or $\beta \ll 1$. Using the perturbation theory find the corrections to the energy levels up to the lowest nonvanishing order.

Appendix 1: Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Appendix 2: Perturbation theory formulae (from lecture)

$$H = H^0 + \lambda H', \qquad E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \dots , \qquad \psi_n = \psi_n^{(0)} + \lambda \psi_n^{(1)} + \lambda^2 \psi_n^{(2)} + \dots$$

$$E_n^{(1)} = H'_{nn}$$

$$\psi_n^{(1)} = \sum_m c_{nm} \psi_m^{(0)}, \qquad c_{nm} = \begin{cases} \frac{H'_{mn}}{E_n^{(0)} - E_m^{(0)}}, & n \neq m \\ 0, & n = m \end{cases}$$

$$E_n^{(2)} = \sum_{m \neq n} \frac{|H'_{mn}|^2}{E_n^{(0)} - E_m^{(0)}}$$

$$\psi_n^{(2)} = \sum_m d_{nm} \psi_m^{(0)}, \qquad d_{nm} = \begin{cases} \frac{1}{E_n^{(0)} - E_m^{(0)}} \left(\sum_{k \neq n} \frac{H'_{mk} H'_{kn}}{E_n^{(0)} - E_k^{(0)}} \right) - \frac{H'_{nn} H'_{mn}}{\left(E_n^{(0)} - E_m^{(0)} \right)^2}, & n \neq m \\ 0, & n = m \end{cases}$$