PHYS 452 Quantum Mechanics II (Fall 2018)
Homework #5, due Thursday November 8 in class

Time-dependent perturbation theory. Quantum dynamics of spin.

1. Consider a hydrogen atom in the ground state ($n = 1$). What is the the probability of transition to $n = 2$ states at $t = +\infty$ when the atom is placed in a uniform electric field that has the following time-dependence:

$$\mathcal{E}(t) = \begin{cases} \mathcal{E}_0 e^{-t/\tau}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

   How small $\mathcal{E}_0$ should be for the calculations to be valid?

2. Consider a quantum harmonic oscillator, initially in the ground state. It is subject to the time-dependent perturbation in the form

$$H'(x,t) = -e\mathcal{E}x \exp(-t^2/\tau^2)$$

   which is the same as was in the example we considered in lecture. Here, e is the elementary charge, $\mathcal{E}$ is the magnitude of the uniform electric field (we assume it to be small), and τ is a constant that defines the pulse duration. Find the probability of transition from the ground state, $|0\rangle$, to the *second* excited state, $|2\rangle$, at $t = +\infty$. Note that this problem demands consideration of the second order of the perturbation theory.

3. An electron is placed in a static uniform magnetic field $\mathbf{B}_0 = (0, 0, B_0)$. Initially, at $t = 0$, it occupies the state with a positive projection of spin on the z-axis. Then, at $t = 0$ an additional time-dependent, spatially uniform, magnetic field $\mathbf{B}_1(t) = (B_1 \cos\omega t, B_1 \sin\omega t, 0)$ is turned on. Calculate the probability of finding the electron in the state with a negative projection of its spin on the z-axis at time $t > 0$. Do not assume that B_1 is small compared to B_0. For convenience, you may introduce two angular frequencies, $\omega_0 = \frac{|e|B_0}{2m}$ and $\omega_1 = \frac{|e|B_1}{2m}$ (here e is the electron charge and m is its mass). This problem has some similarity with the dynamics of a two-level atom that we considered in lecture.