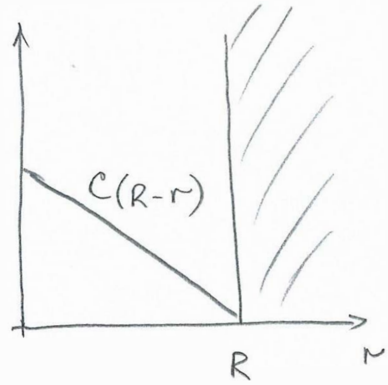
Since the wave function must vanish at the well boundary ($r = R$), our trial wave function can be written as




$$\psi(r) = \begin{cases} C(R-r), & r<b \\ 0, & r>b \end{cases}$$

This trial wave function does not have any free adjustable parameters because the value of the constant $C$ comes from the normalization condition:

$$1 = \int |\psi|^2 d\vec{r} = |C|^2 4\pi \int_0^R (R-r)^2 r^2 dr = |C|^2 4\pi \int_0^R (r^4 - 2Rr^3 + R^2 r^2) dr =$$

$$= |C|^2 4\pi \left( \frac{R^5}{5} - \frac{1}{2}R^5 + \frac{1}{3}R^5 \right) = |C|^2 \frac{2\pi}{15} R^2 \quad \text{or} \quad |C|^2 = \frac{15}{2\pi R^5}$$

The expectation value of the Hamiltonian is:

$$E = \langle H \rangle = \int \psi \left( -\frac{\hbar^2}{2m} \nabla^2 \right) \psi \, d\vec{r} = -|C|^2 \frac{\hbar^2}{2m} 4\pi \int_0^R (R-r) \left[ \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} (R-r) \right] r^2 dr$$

$$= \frac{15}{\cancel{2\pi} R^5} \frac{\hbar^2 \cdot \cancel{4\pi}}{\cancel{2m}} \int_0^R (R-r) \left[ \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \right] r^2 dr = \frac{15\hbar^2}{mR^5} 2 \int_0^R (R-r) r \, dr =$$

$$= \frac{30\hbar^2}{mR^5} \left( \frac{R^3}{2} - \frac{R^3}{3} \right) = \frac{5\hbar^2}{mR^2}$$